# Segment Watchman Routes

Anna Brötzner, Omrit Filtser, Bengt J. Nilsson, Christian Rieck, **Christiane Schmidt**

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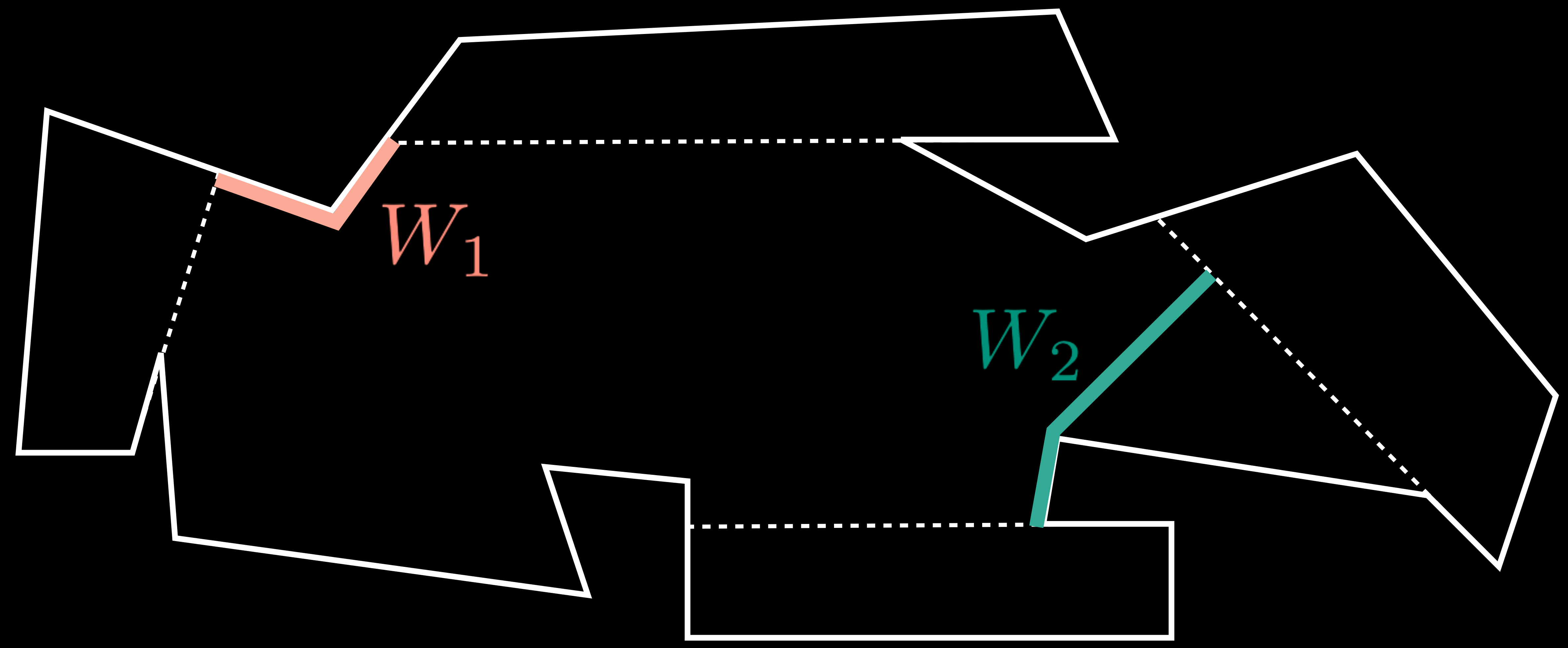


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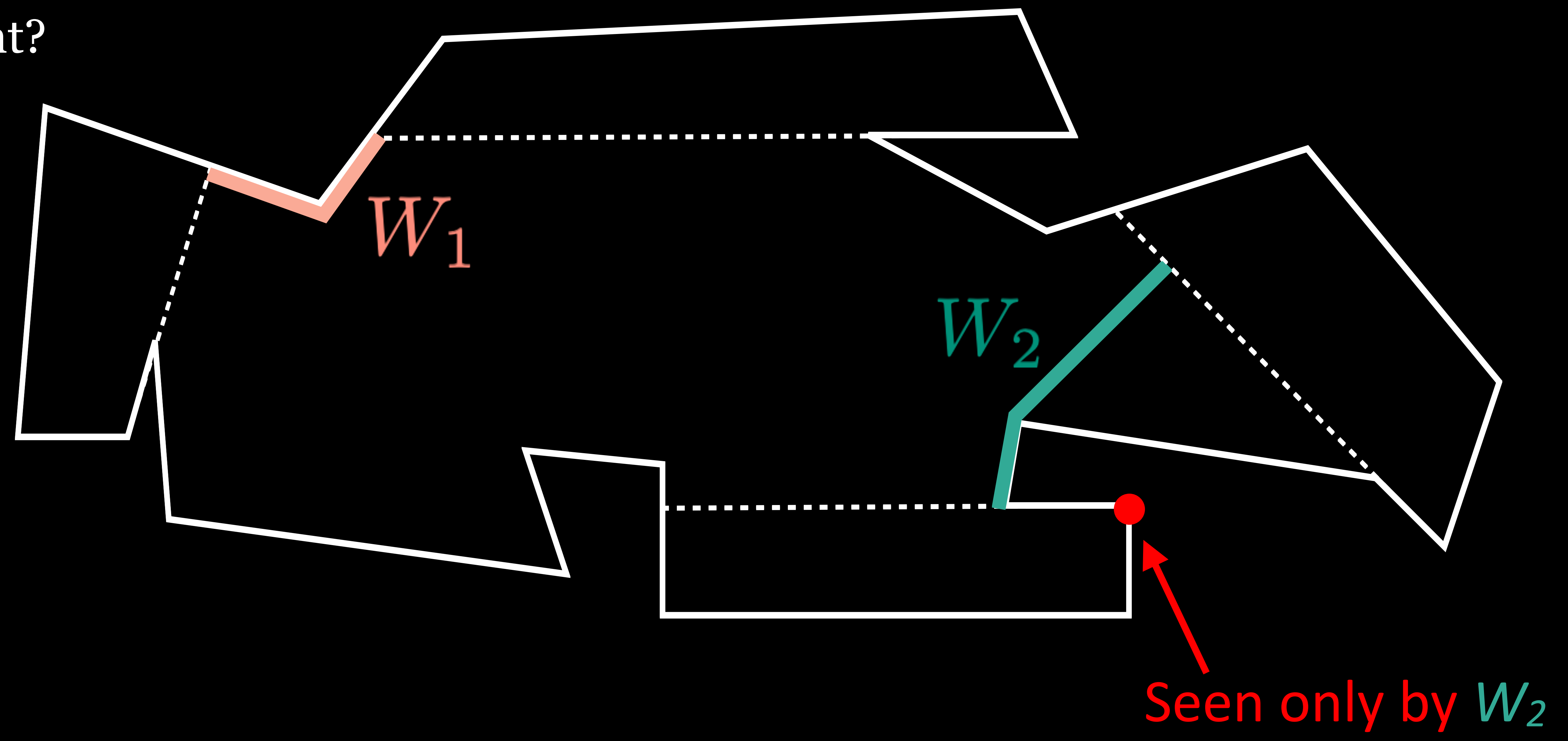


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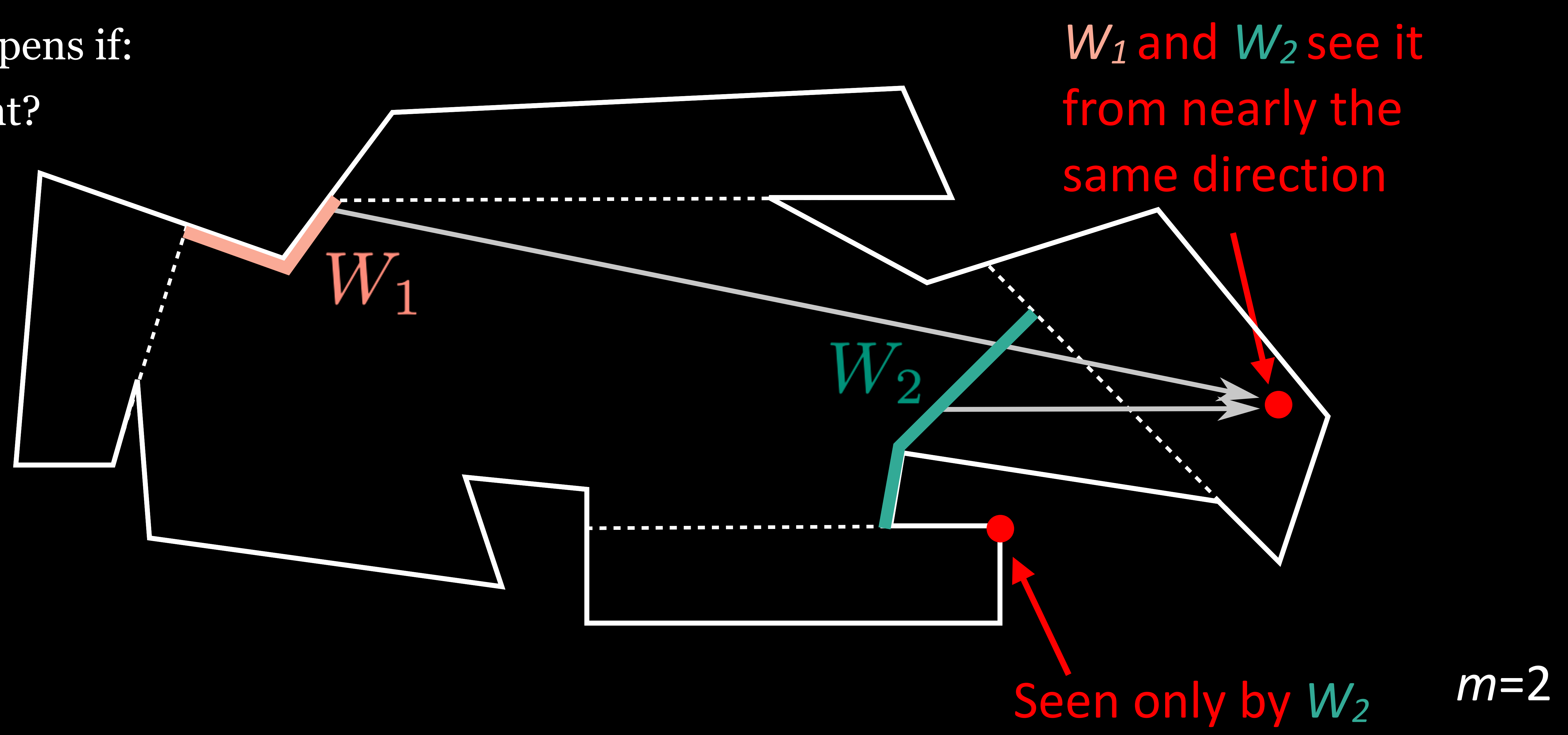

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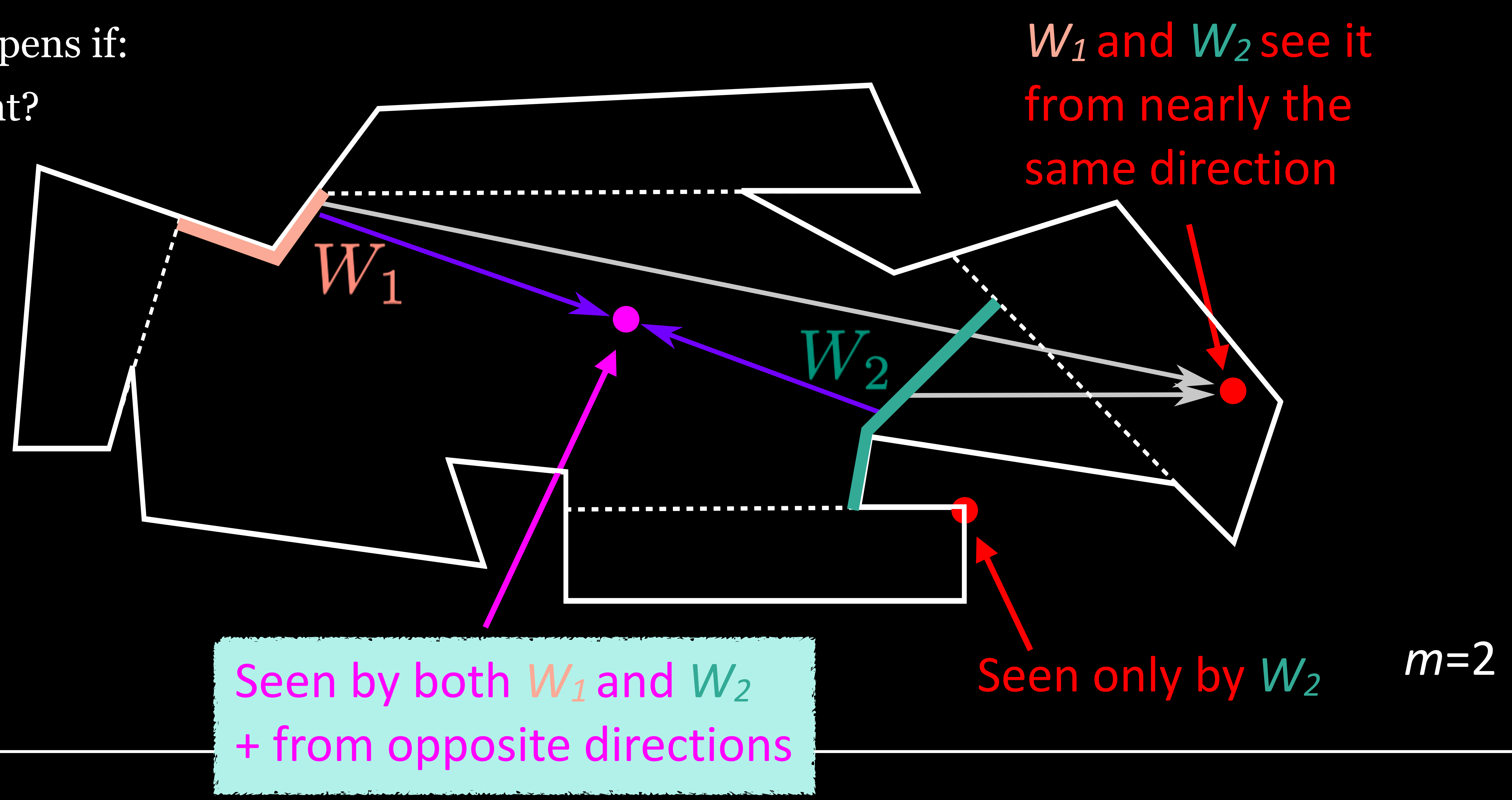

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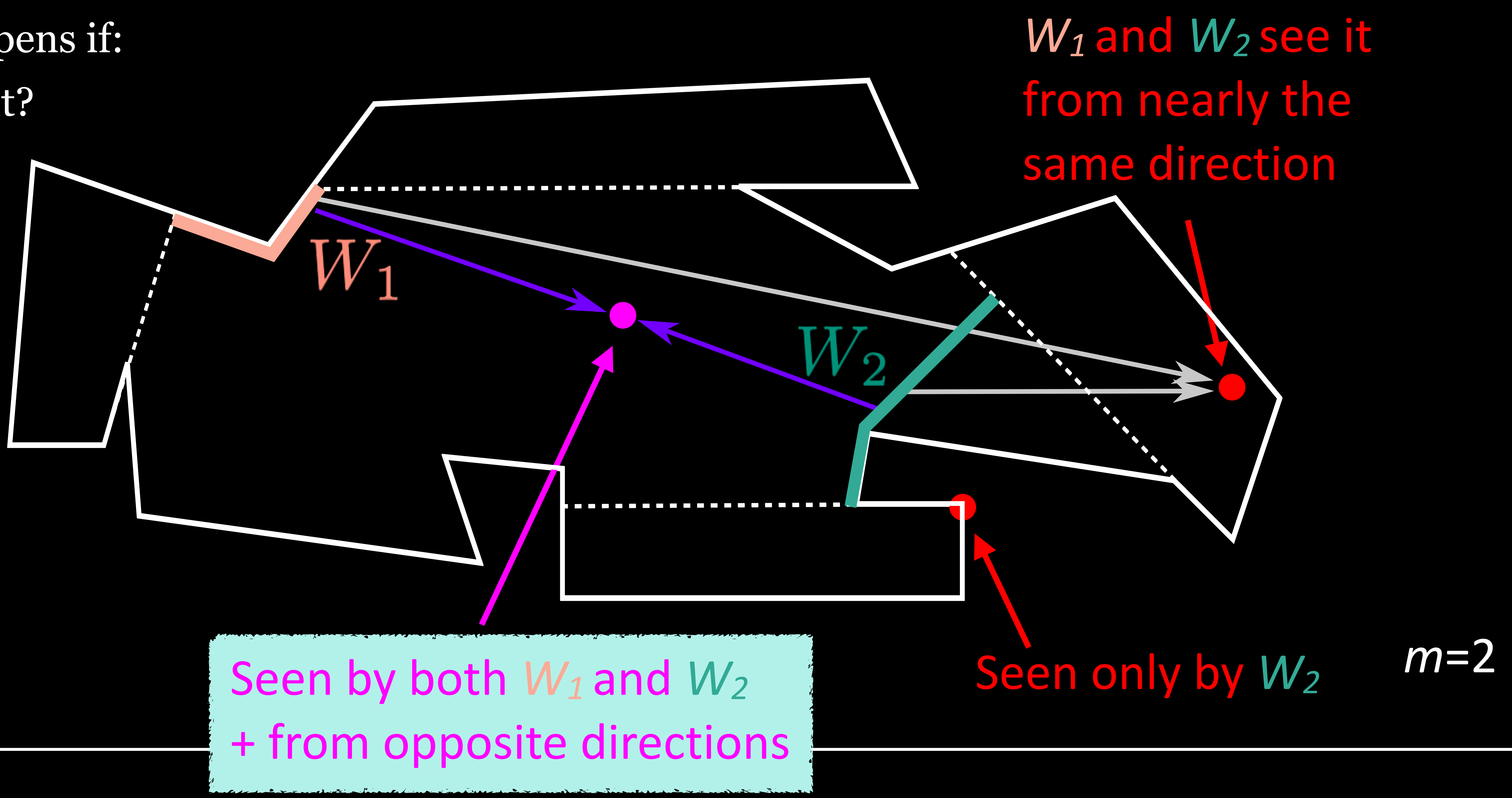
➡ We want to make our routes robust against some of these aspects!



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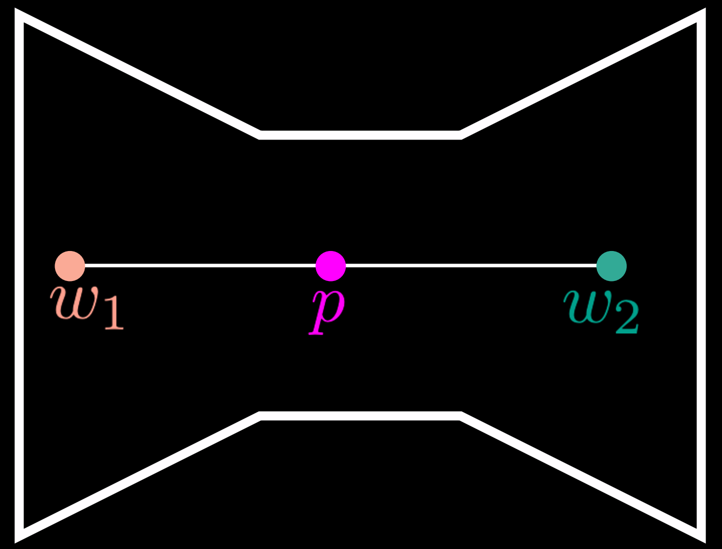

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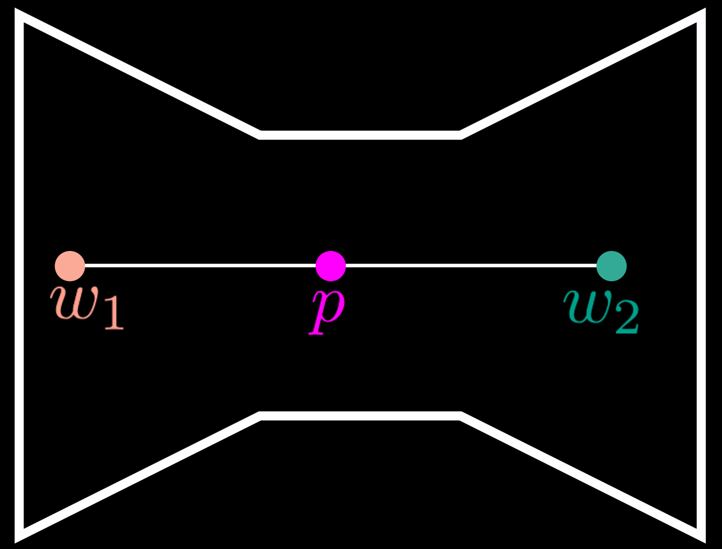


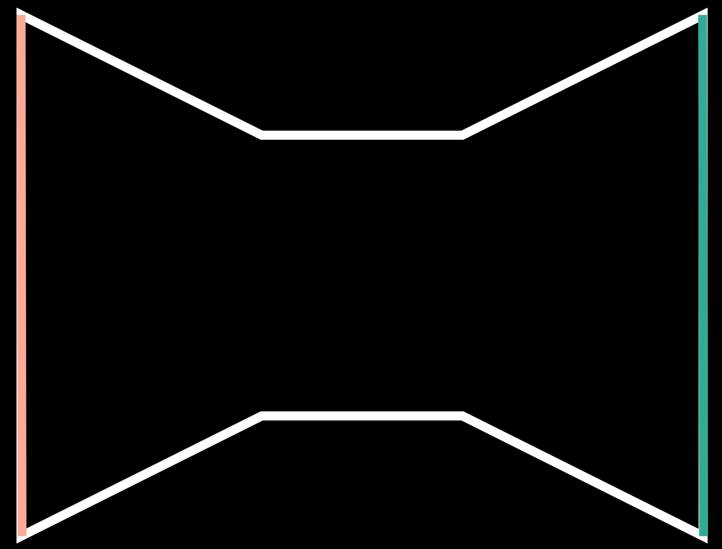

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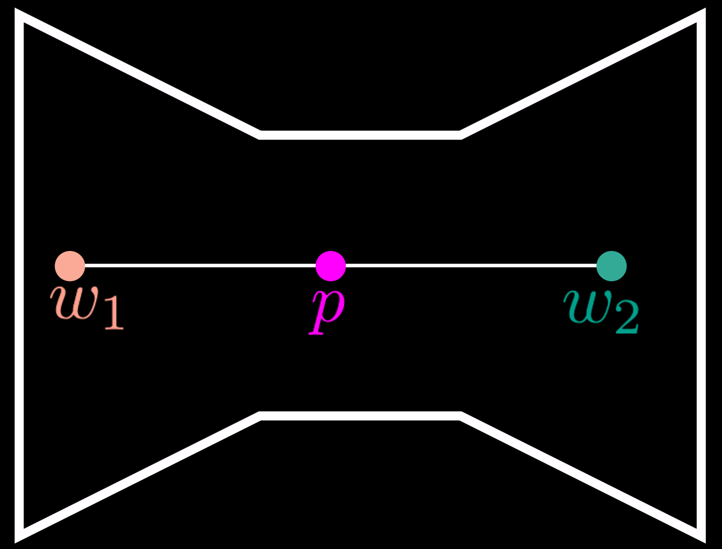


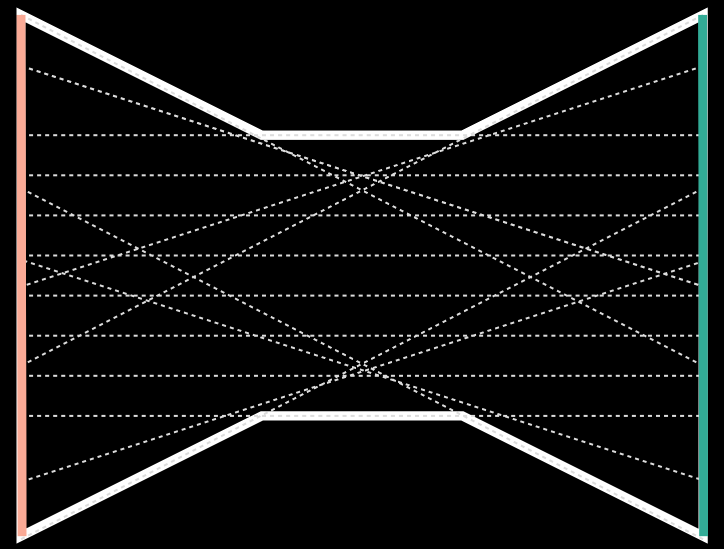

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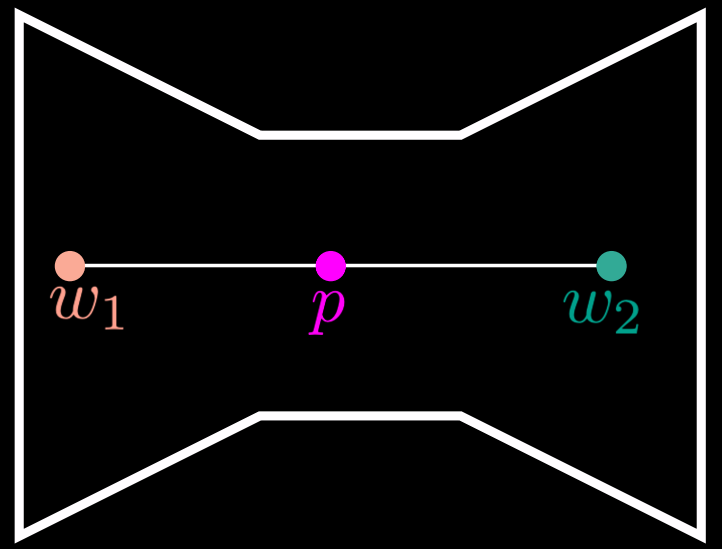


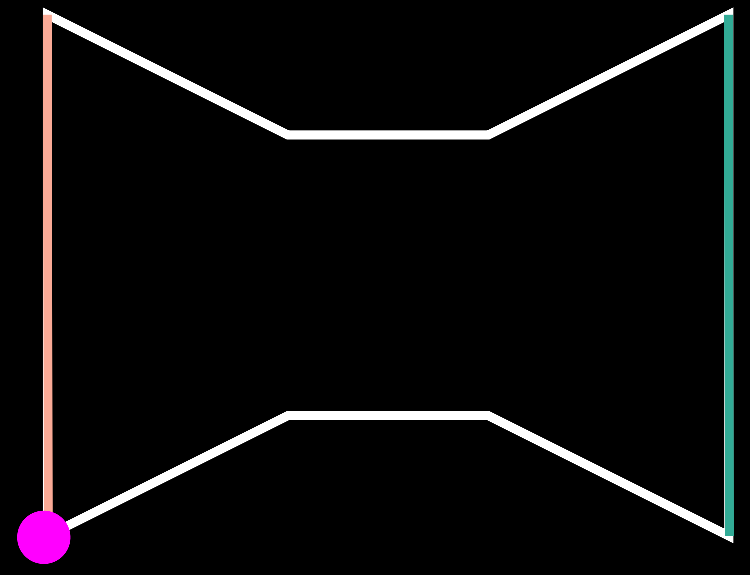

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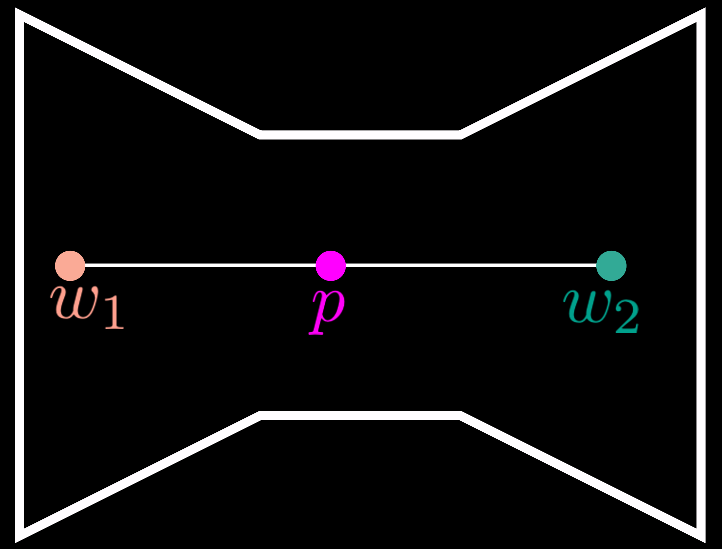


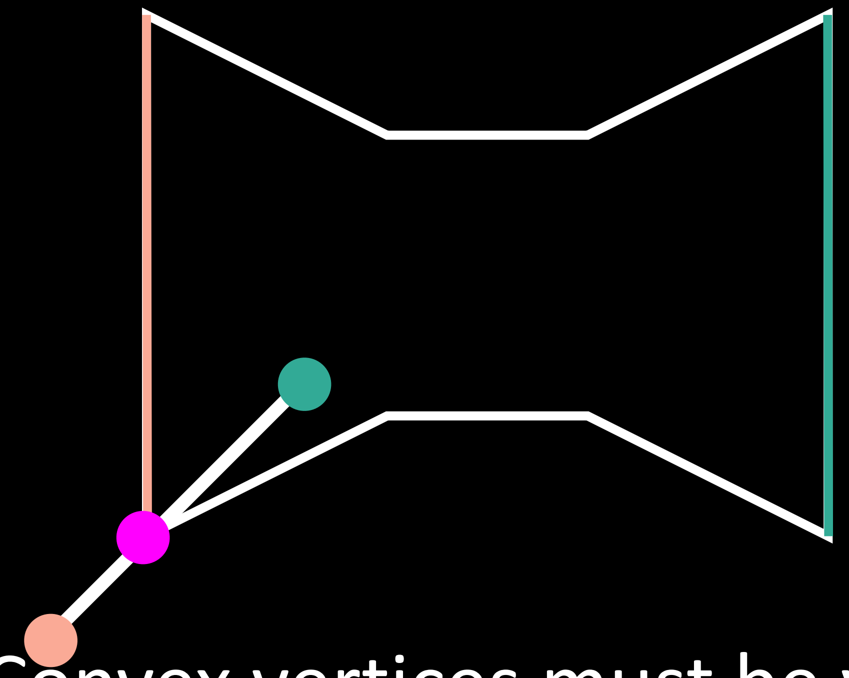

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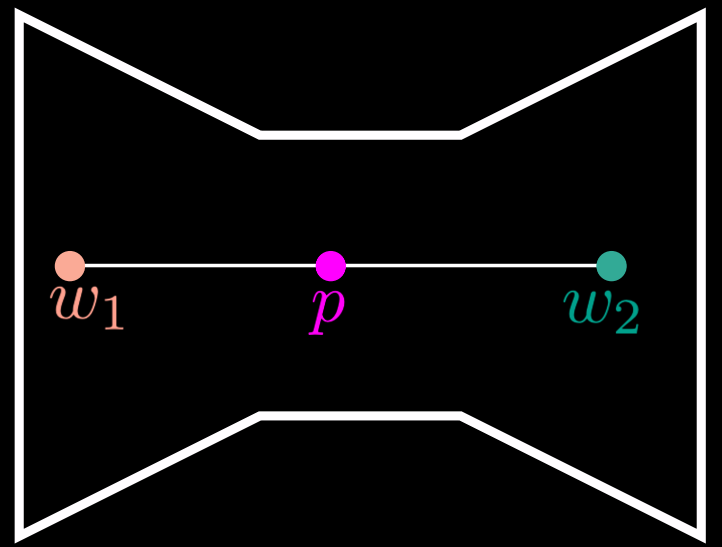


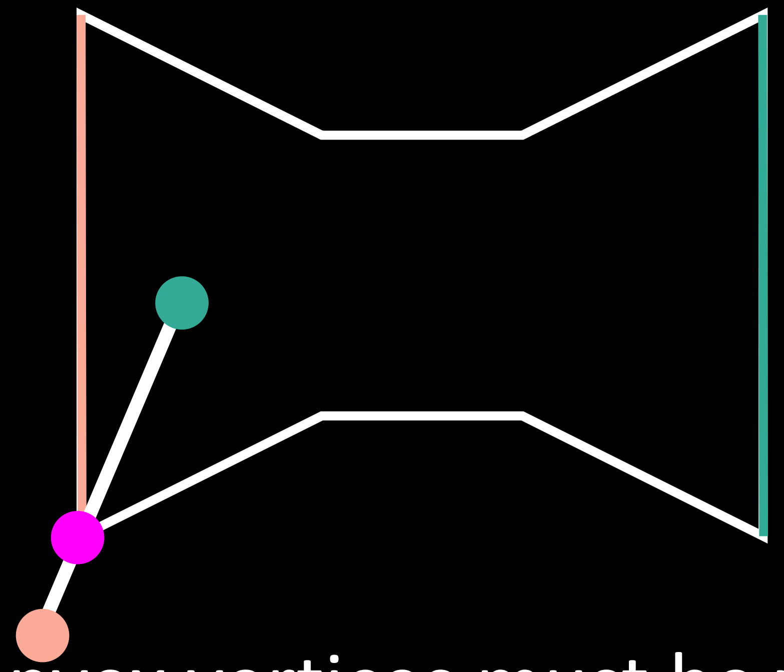

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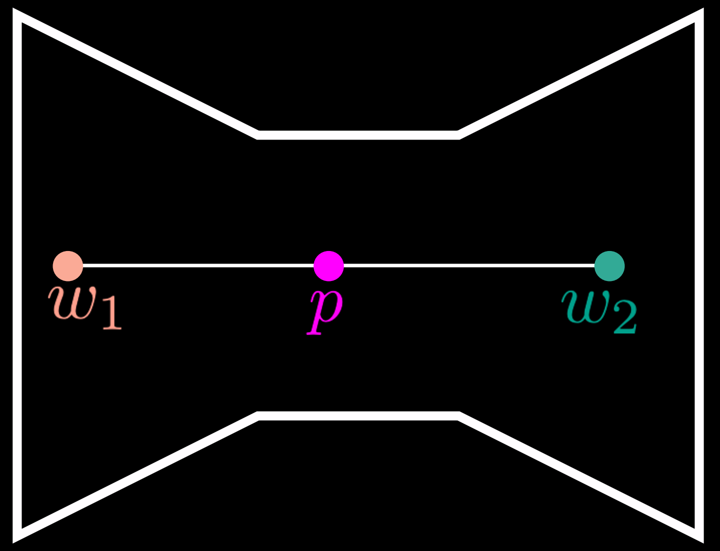


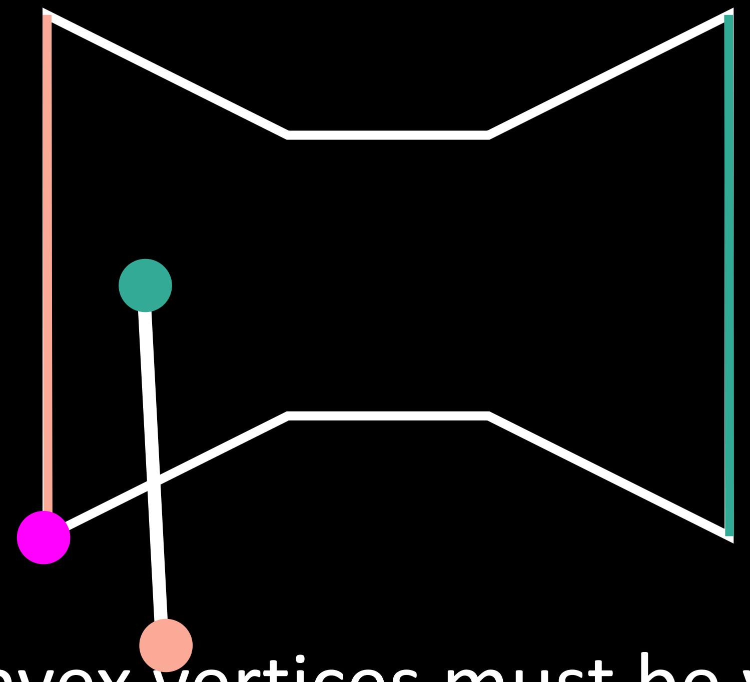

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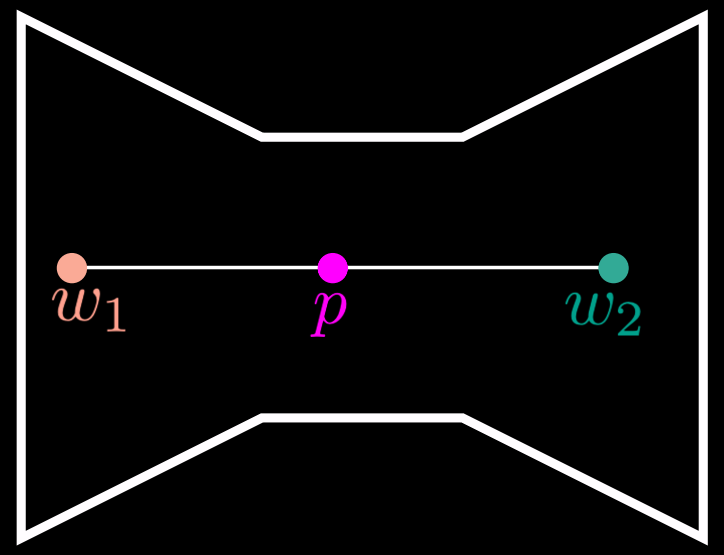


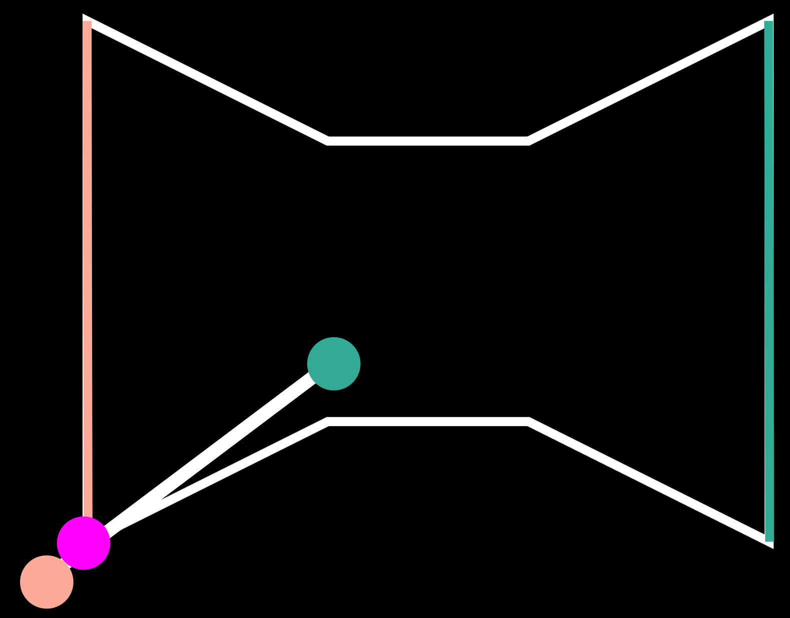

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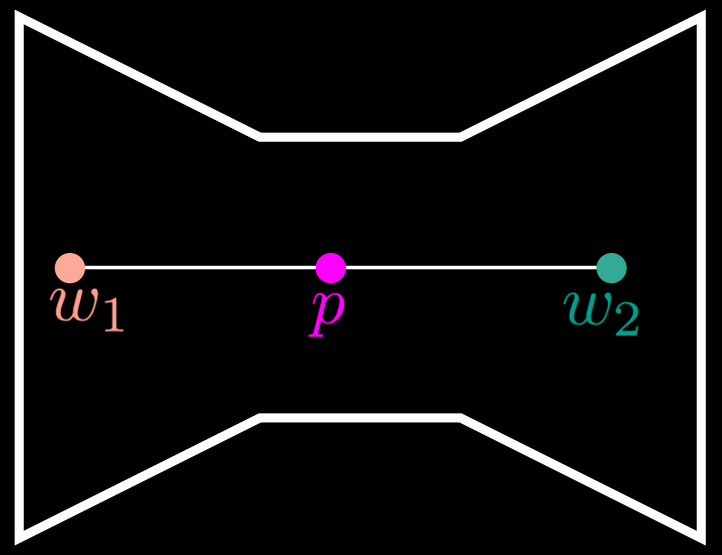


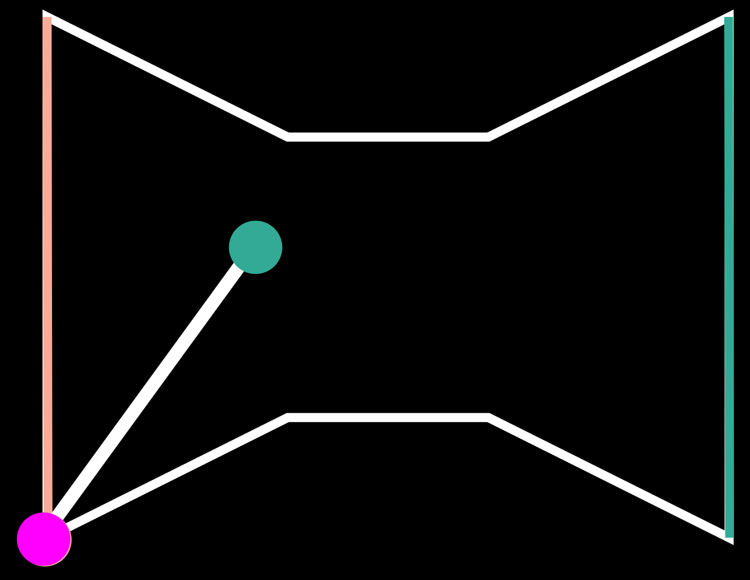

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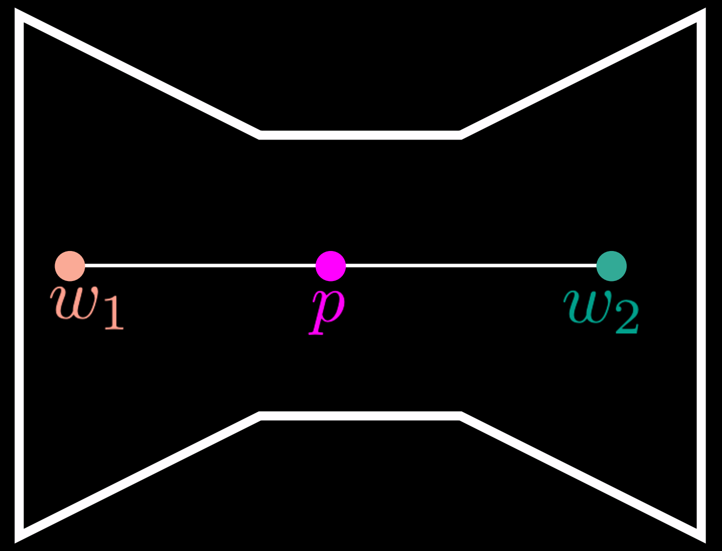


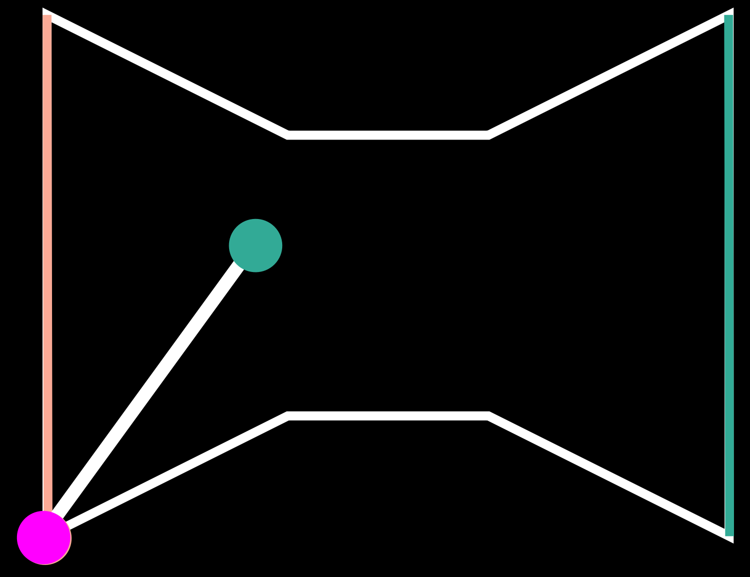

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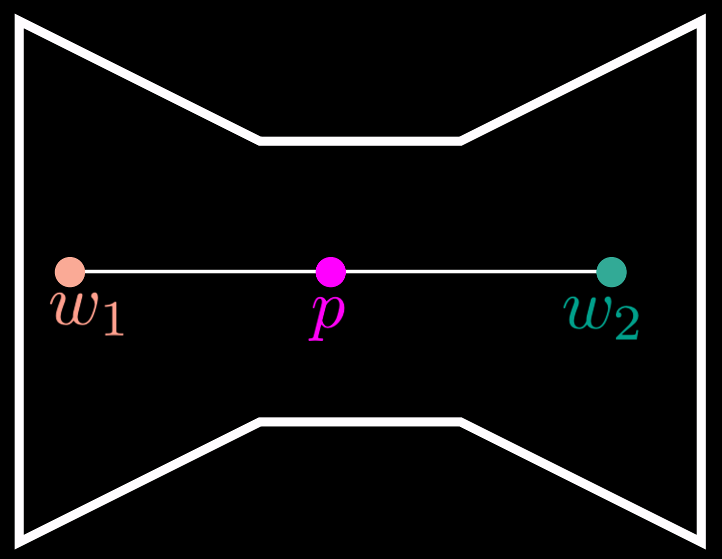


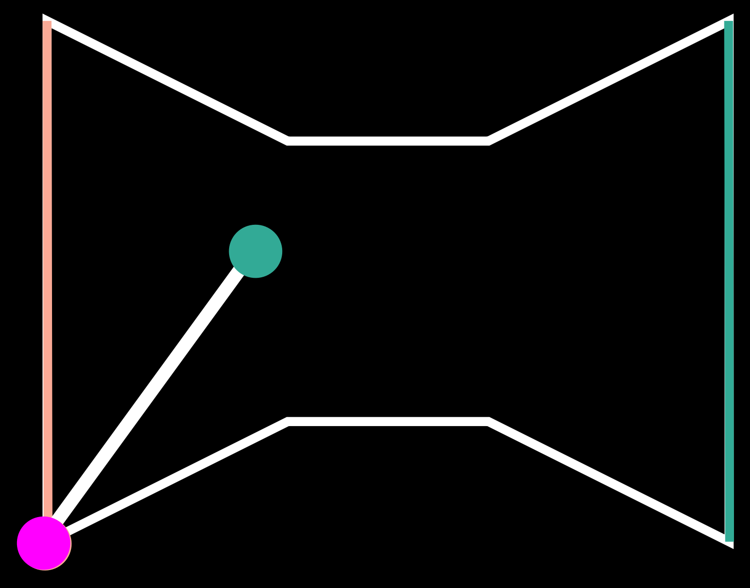

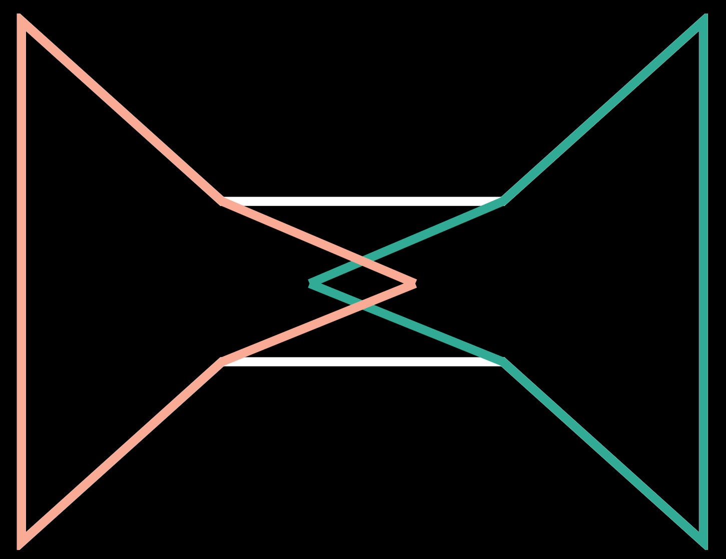

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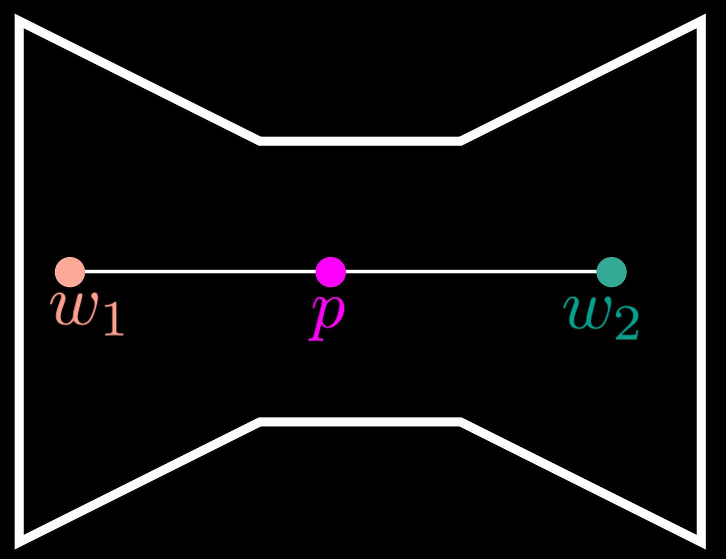


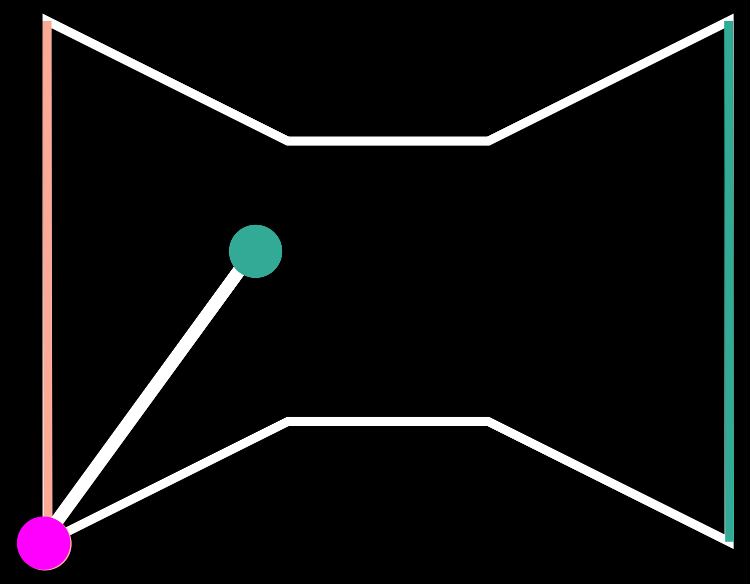

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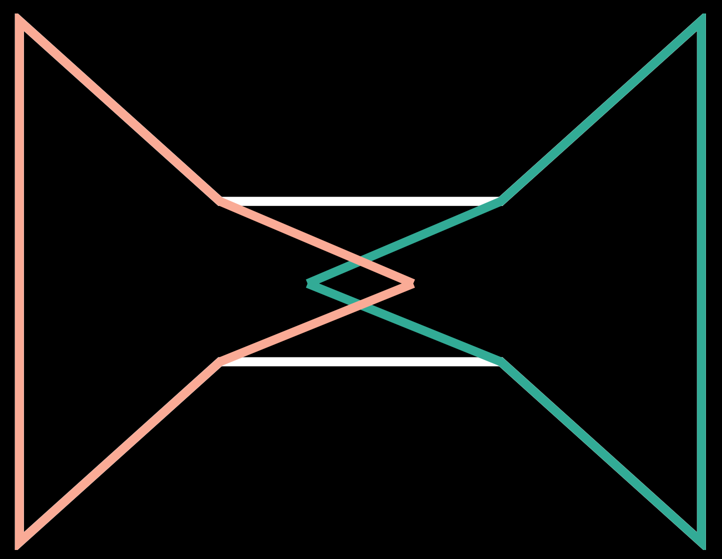

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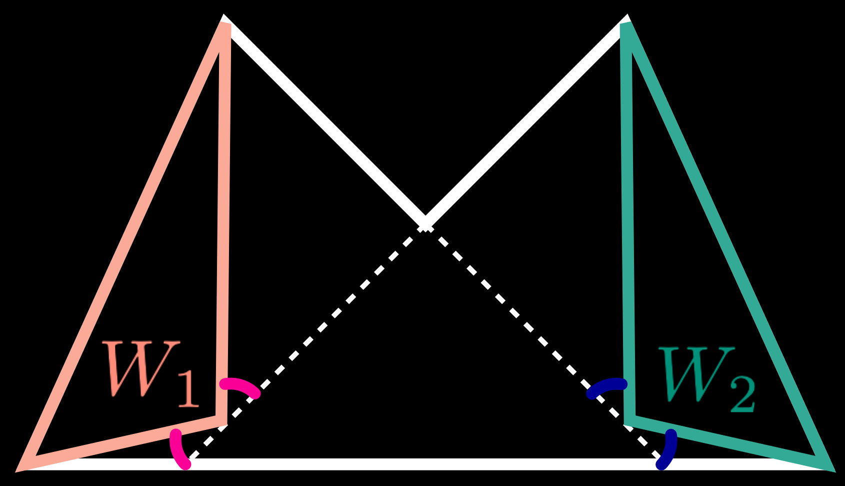


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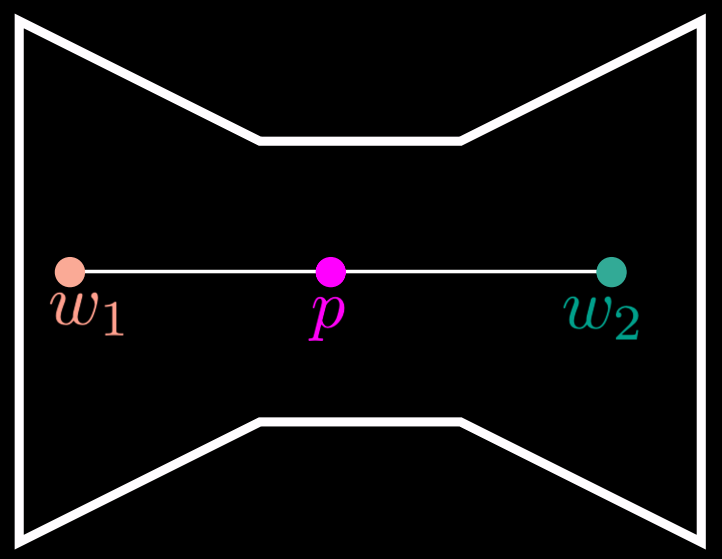


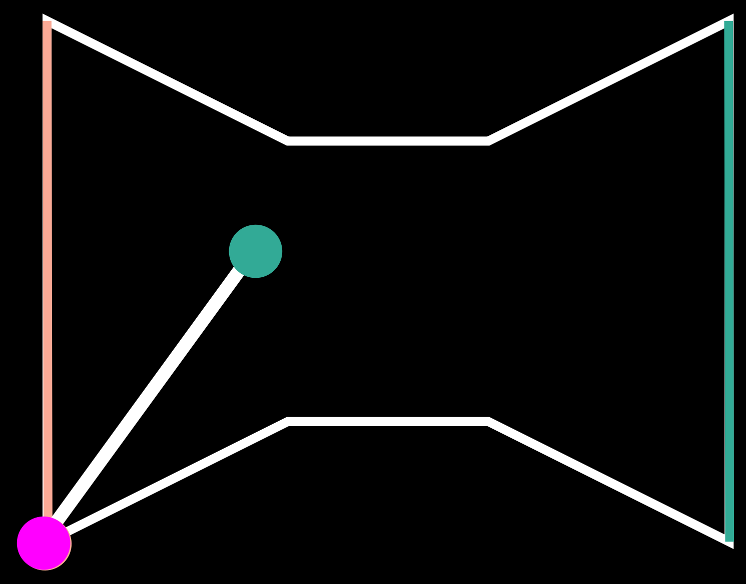

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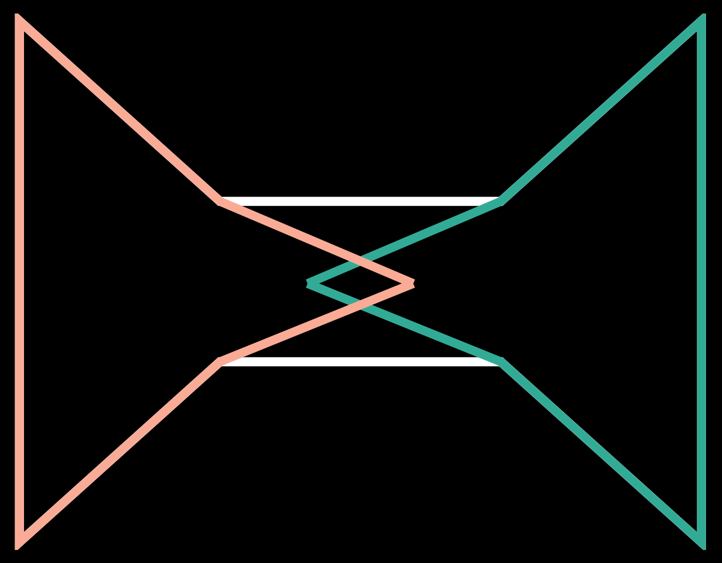

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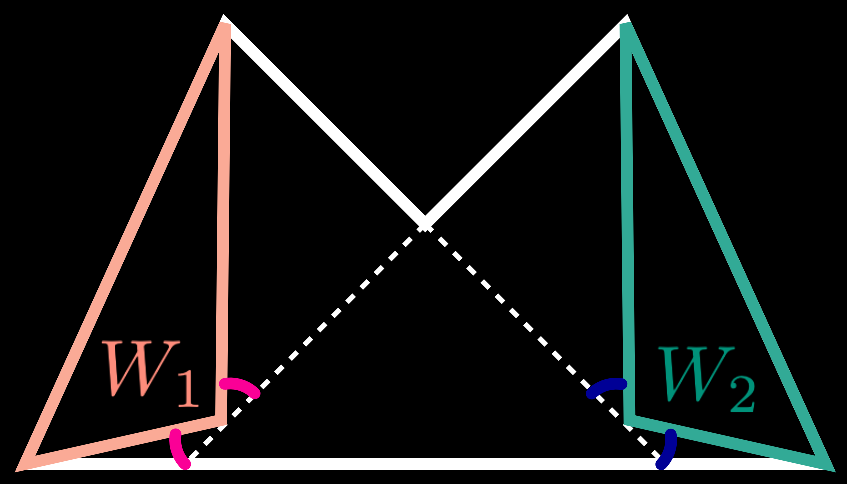


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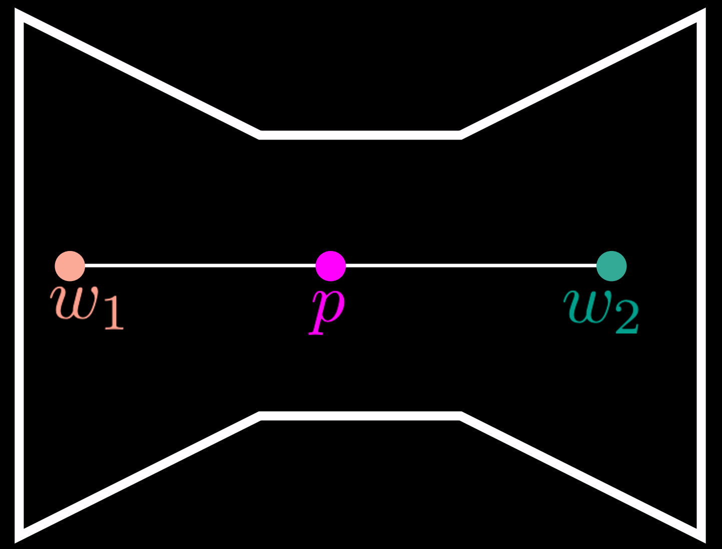
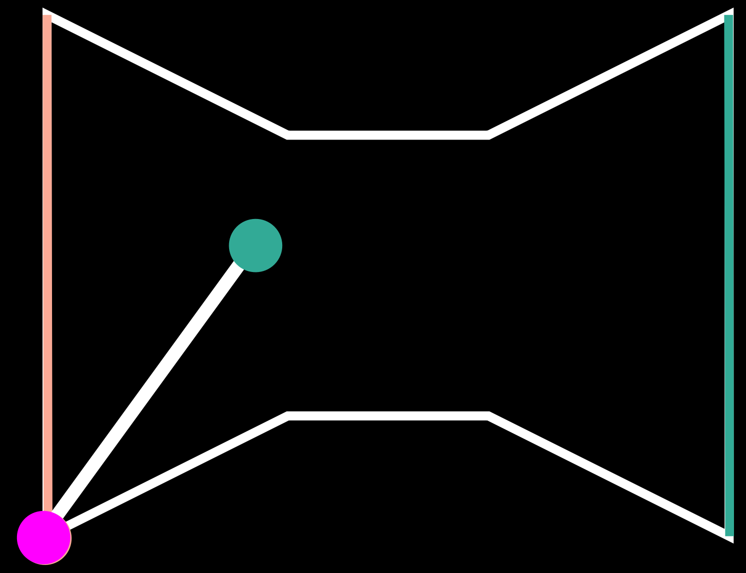
Objectives? Still min-max or min-sum



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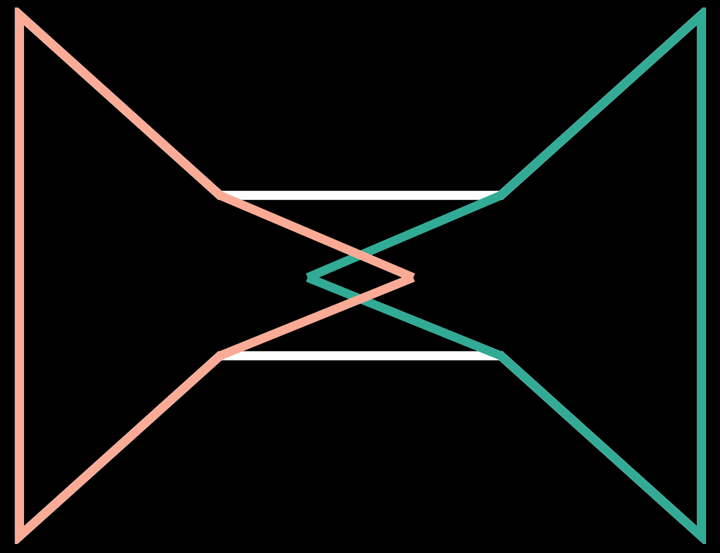

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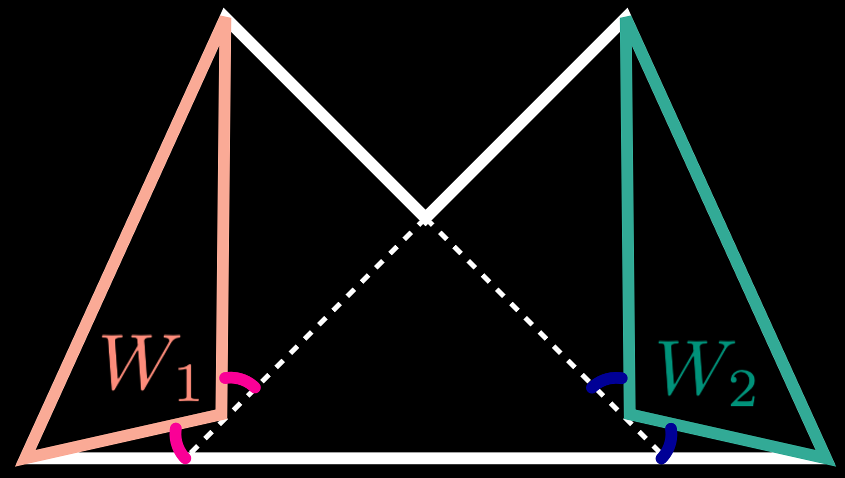


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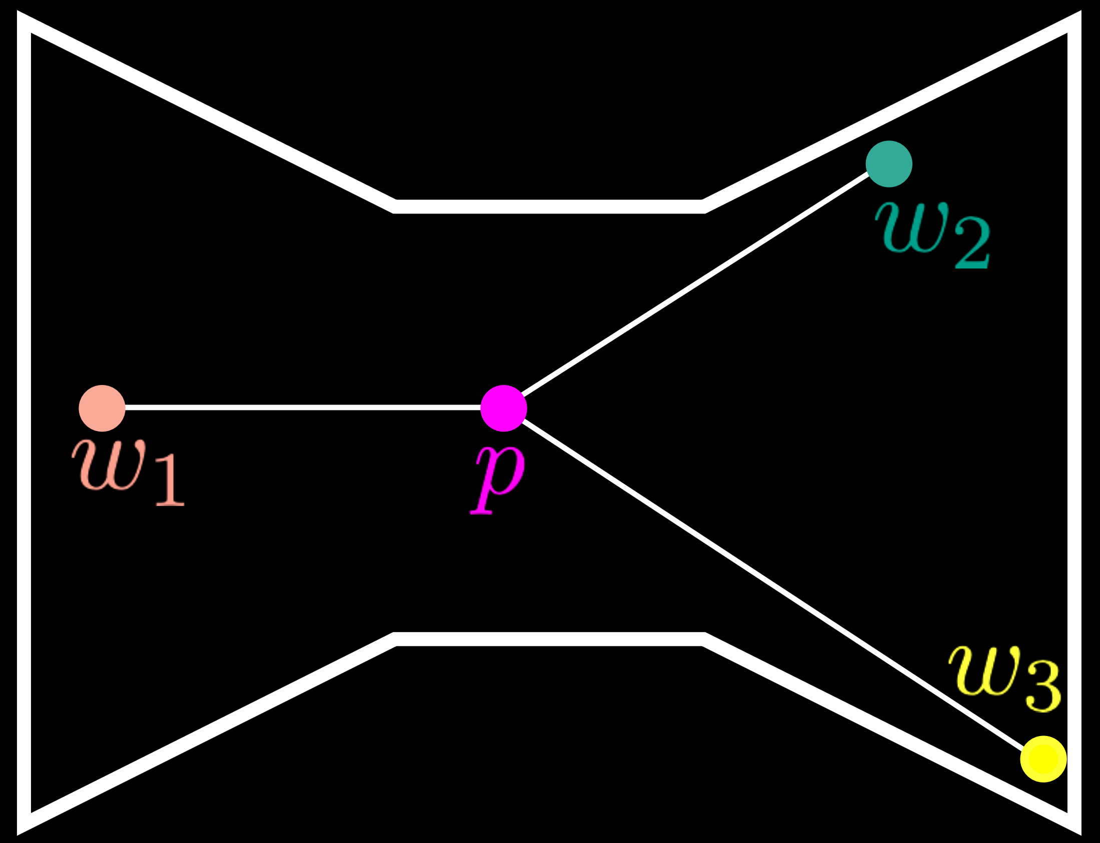
Can be generalized to:

- Triangle-guarded points



Goal: Make $m$-watchman routes robust (today $m$=2)

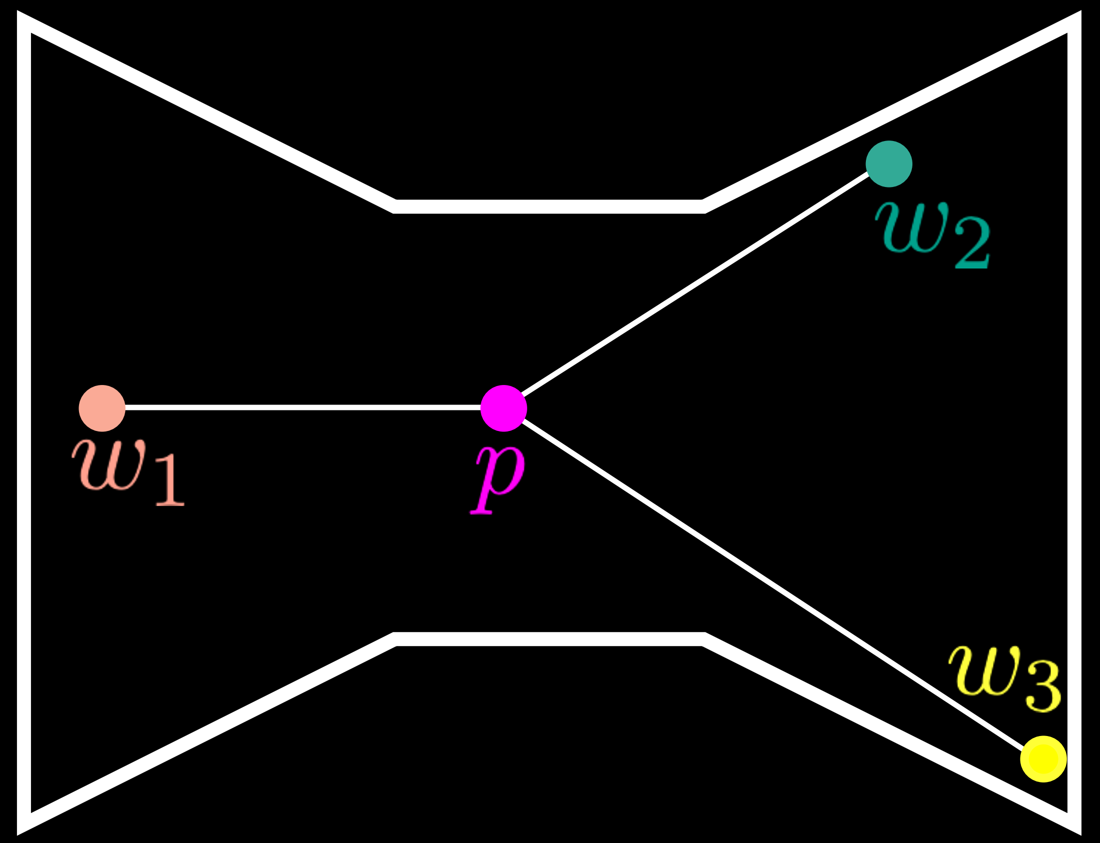
A point $p$ is *segment-guarded* by two points $w_1$ and $w_2$ in the polygon, if:

- $p$ lies on the segment $\overline{w_1 w_2}$
- $p$ is visible from $w_1$ and $w_2$ ($\overline{w_1 w_2}$ fully contained in $P$)

Two routes $W_1$ and $W_2$ are *segment watchman routes* if for every point in the polygon, there exists $w_1 \in W_1$ and $w_2 \in W_2$, such that $p$ is segment-guarded by $w_1$ and $w_2$.

Can be generalized to:

- Triangle-guarded points
- $k$-gon-guarded points



# Sufficient Conditions: The “Conditions Lemma”

*For any two points inside the region enclosed by the route, their shortest path is also contained within the enclosed region

# Sufficient Conditions: The "Conditions Lemma"

Two routes $W_1$ and $W_2$ are segment watchman routes for $P$ if the following conditions hold:

1. Every convex vertex is visited by one of $W_1$ or $W_2$.
2. Both $W_1$ and $W_2$ visit the visibility polygon of each convex vertex.
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Proof: First show (2)$\Rightarrow$ $W_1$ and $W_2$ are watchman routes

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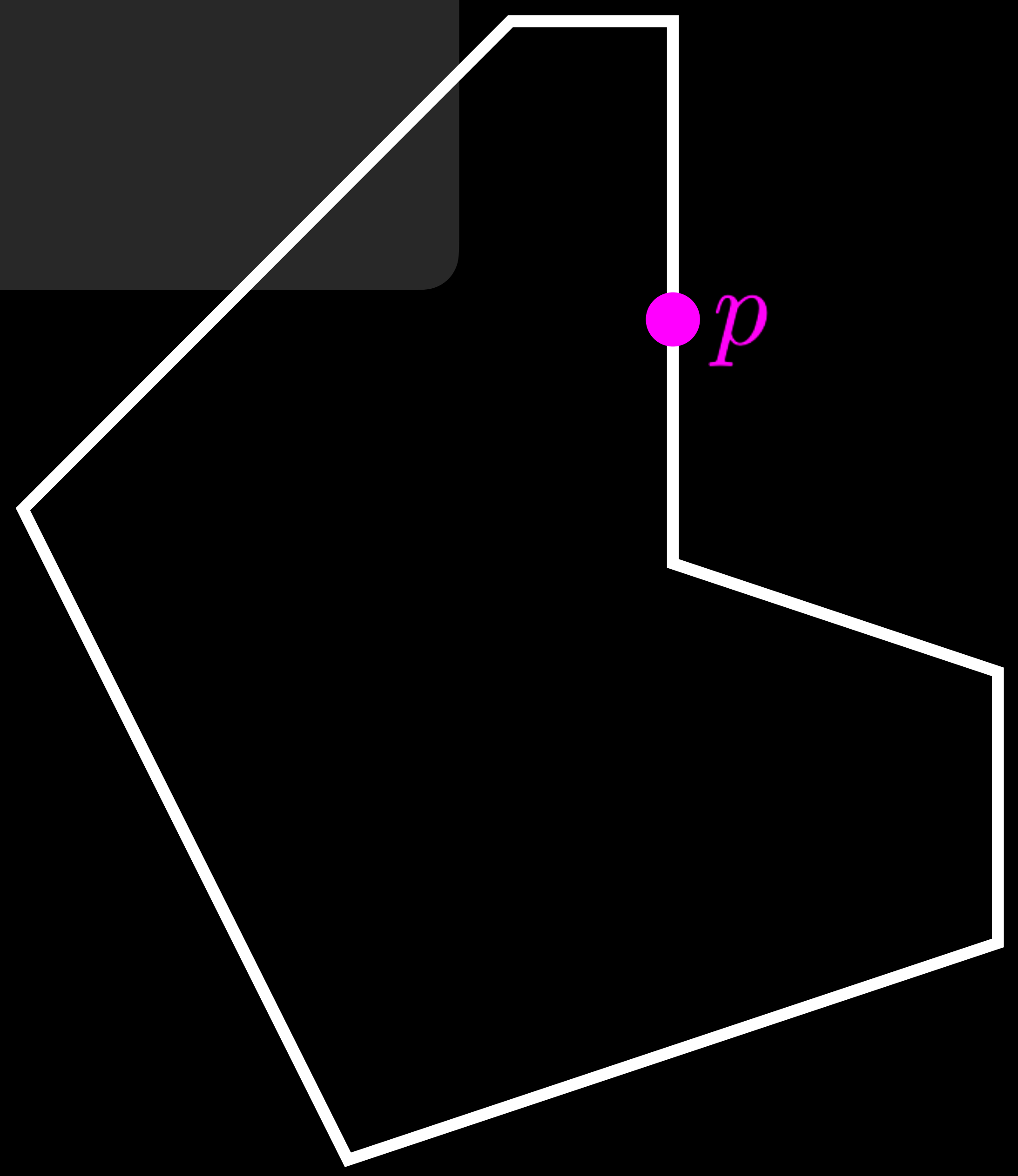

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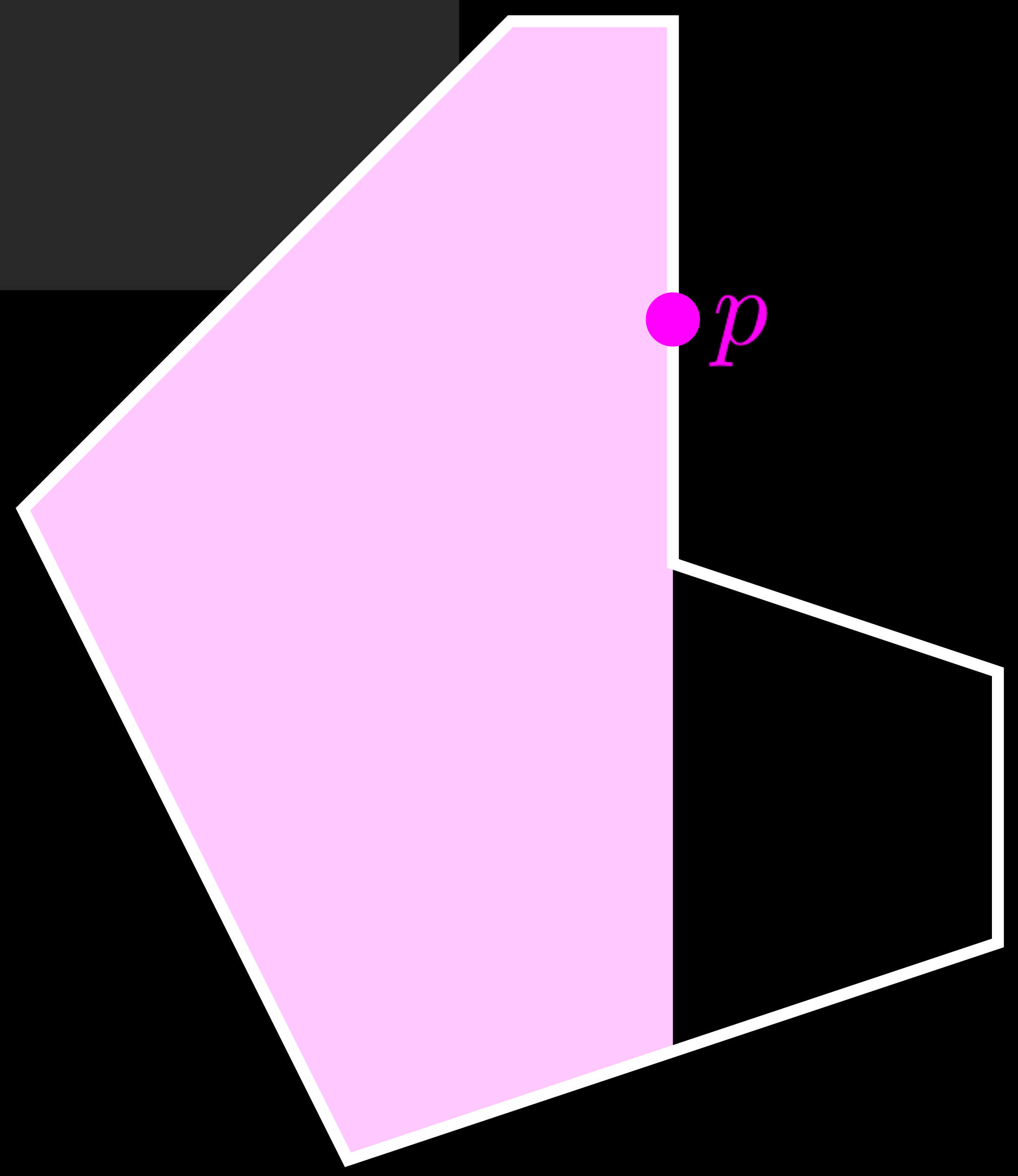


Proof: First show (2)⇒ $W_1$ and $W_2$ are watchman routes

Assume $p \in P$ is not seen by $W_i$

➜ $W_i$ is fully contained in a pocket $P'$ of $p$'s visibility polygon.

Subpolygon of $P$, no point visible from $p$

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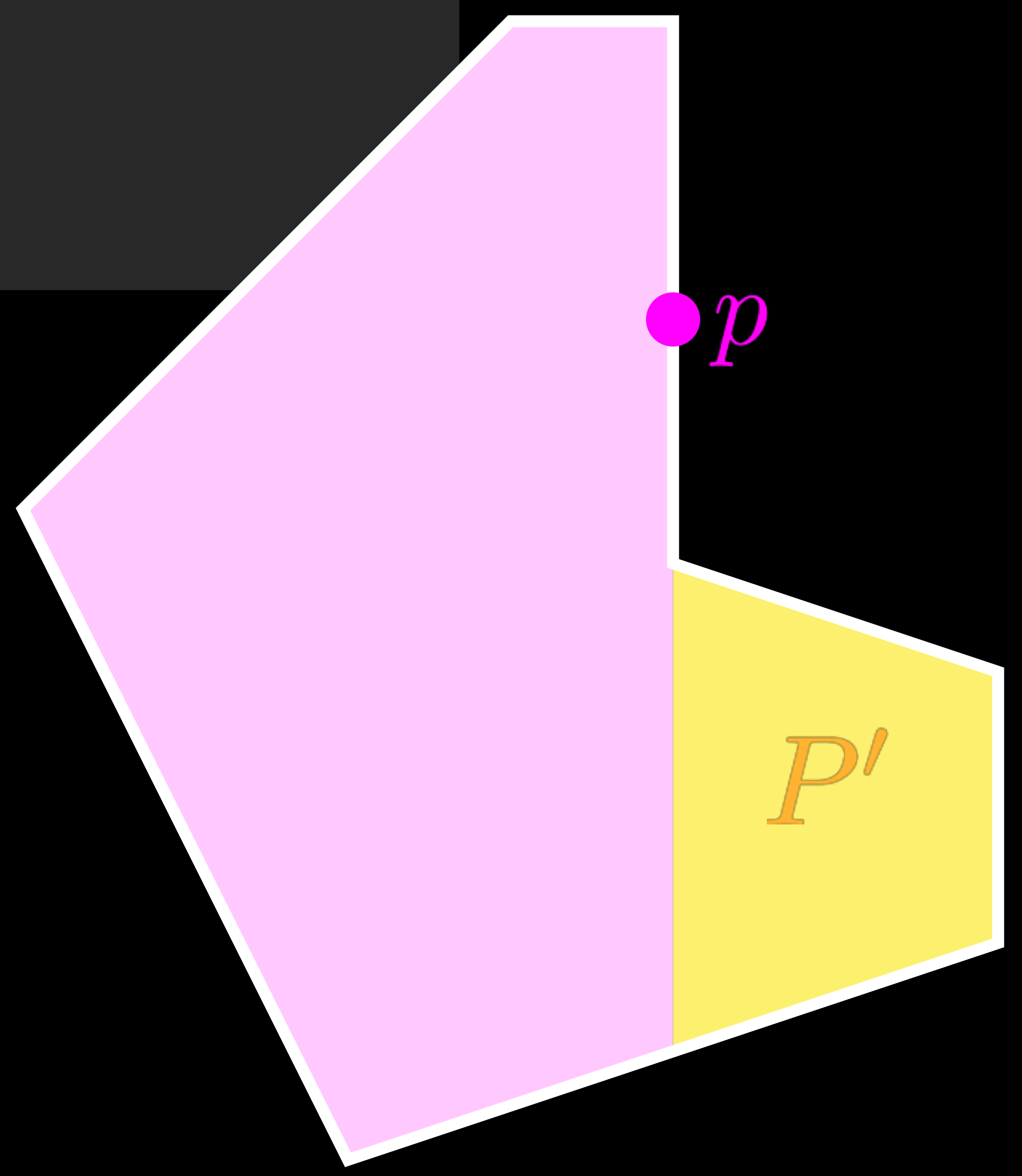


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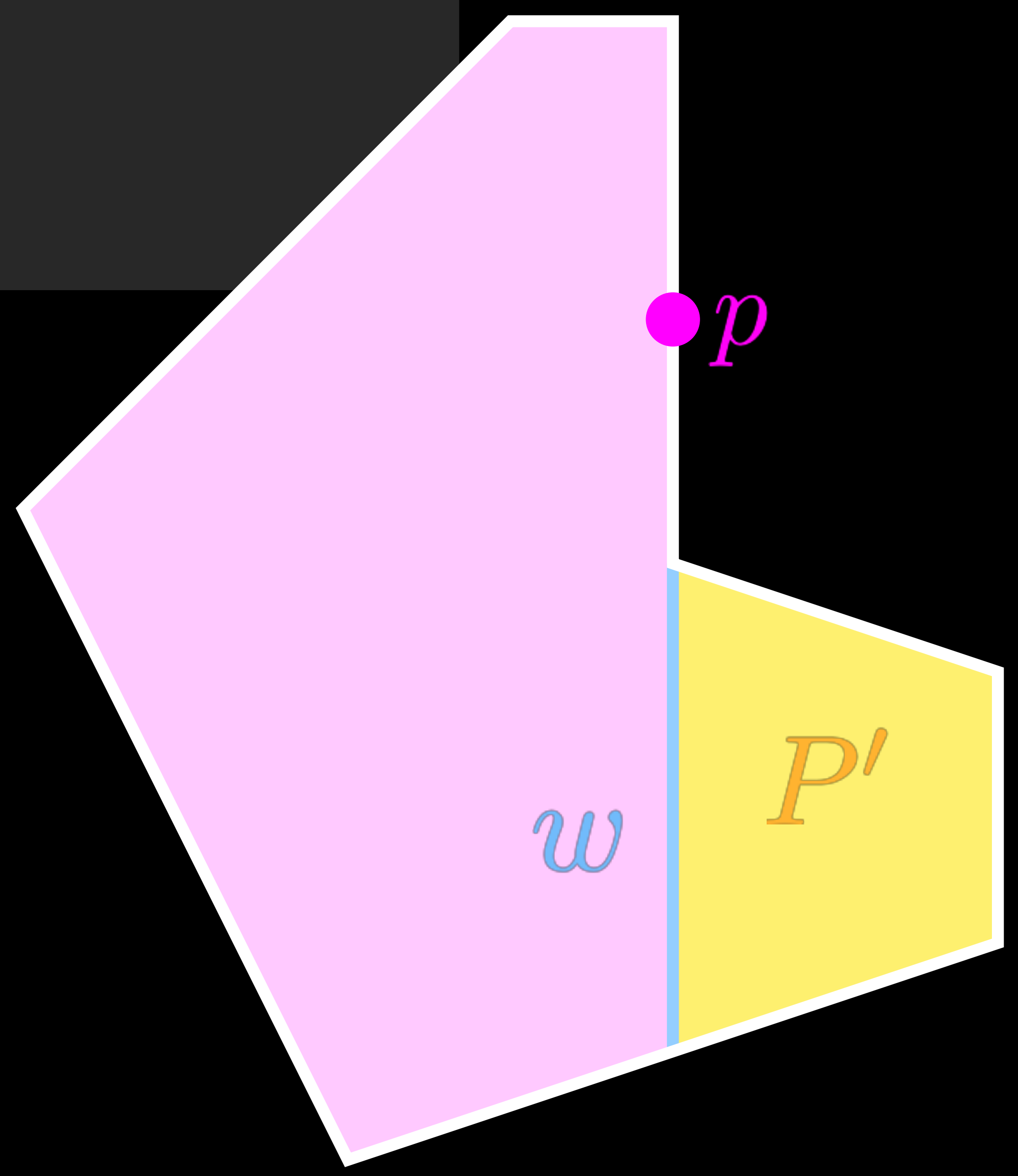
Extend window $w$ of the pocket into a maximal line segment $\ell$




Subpolygon of $P$, no point visible from $p$

Line segment separating $P'$ and $P \setminus P'$

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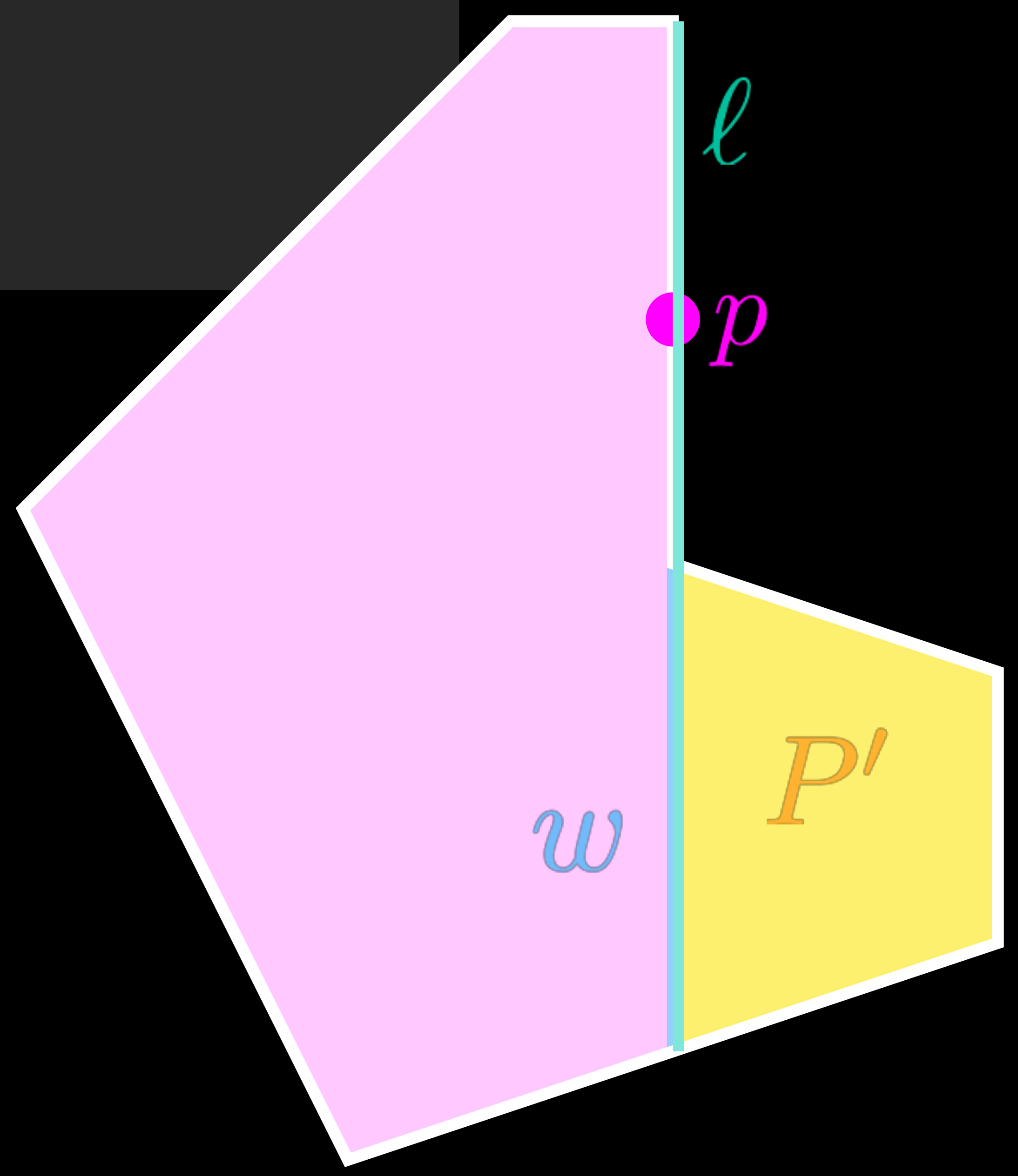


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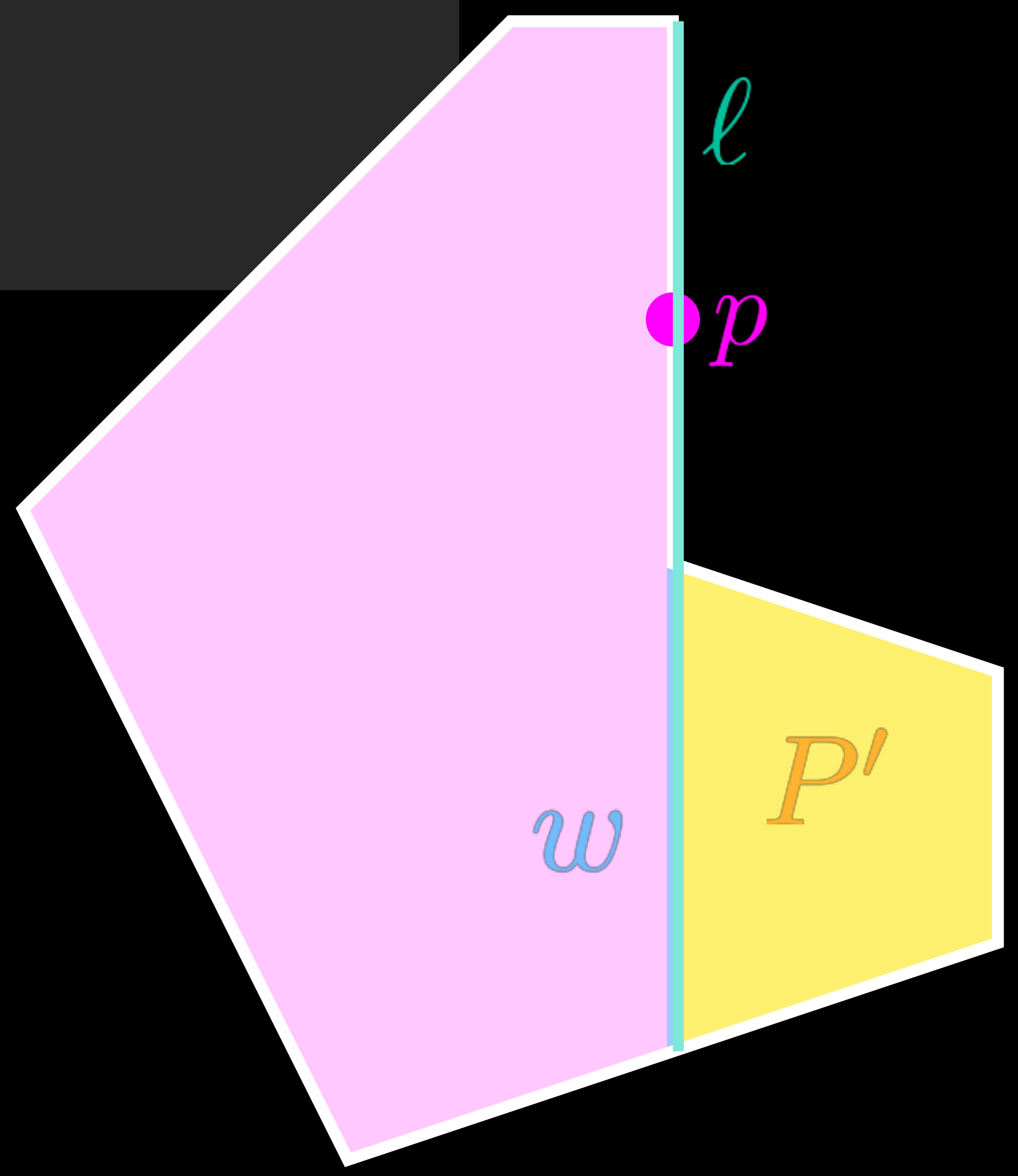


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Extend window $w$ of the pocket into a maximal line segment $\ell$

We know: $p \in \ell$ ➜ Segment $\ell \backslash w$ :

Subpolygon of $P$, no point visible from $p$

Line segment separating $P'$ and $P \backslash P'$

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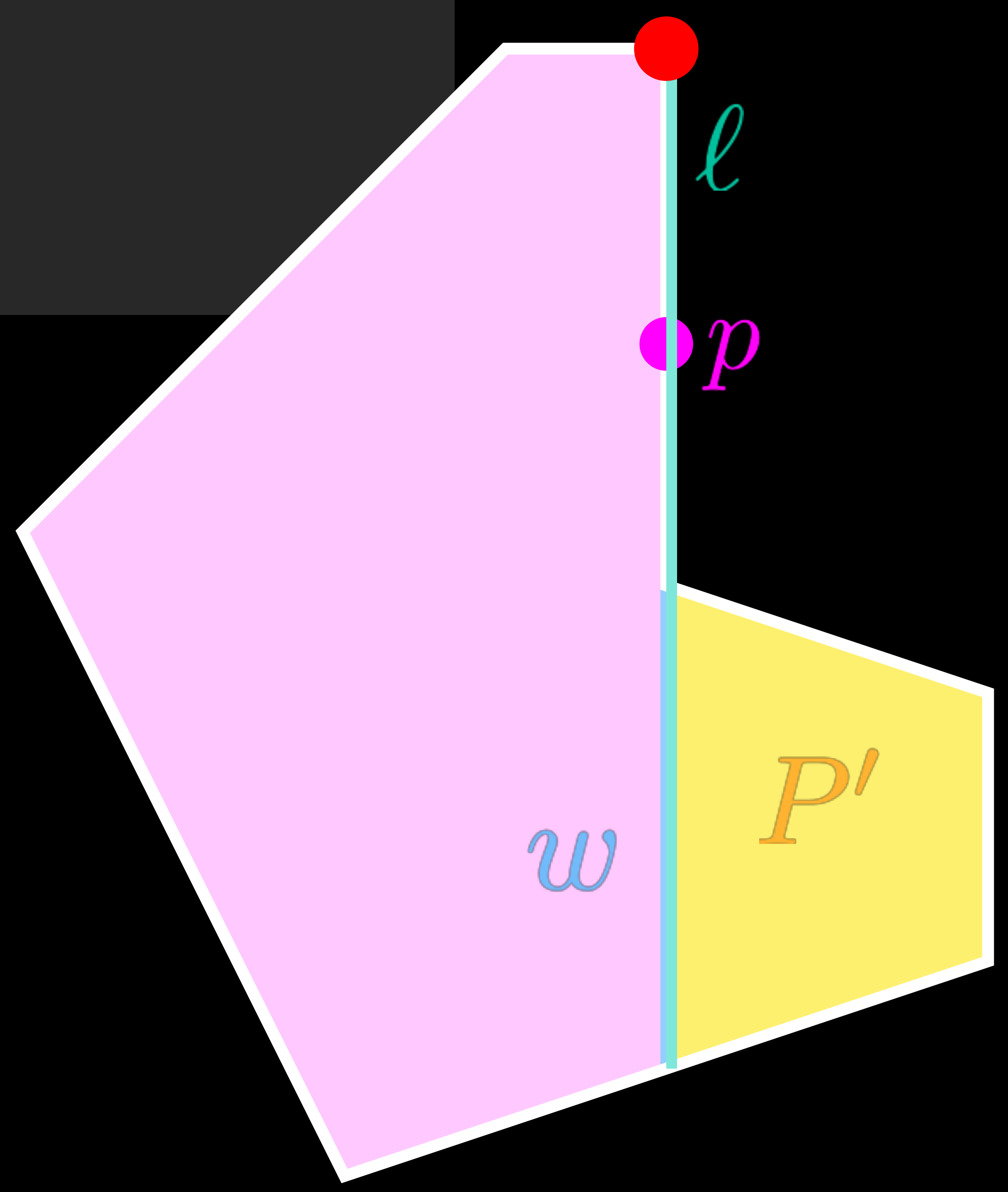


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Extend window $w$ of the pocket into a maximal line segment $\ell$

We know: $p \in \ell$ ➜ Segment $\ell \setminus w$ :

- Is polygonal edge with a convex endpoint not seen by $W_i$

Subpolygon of $P$, no point visible from $p$

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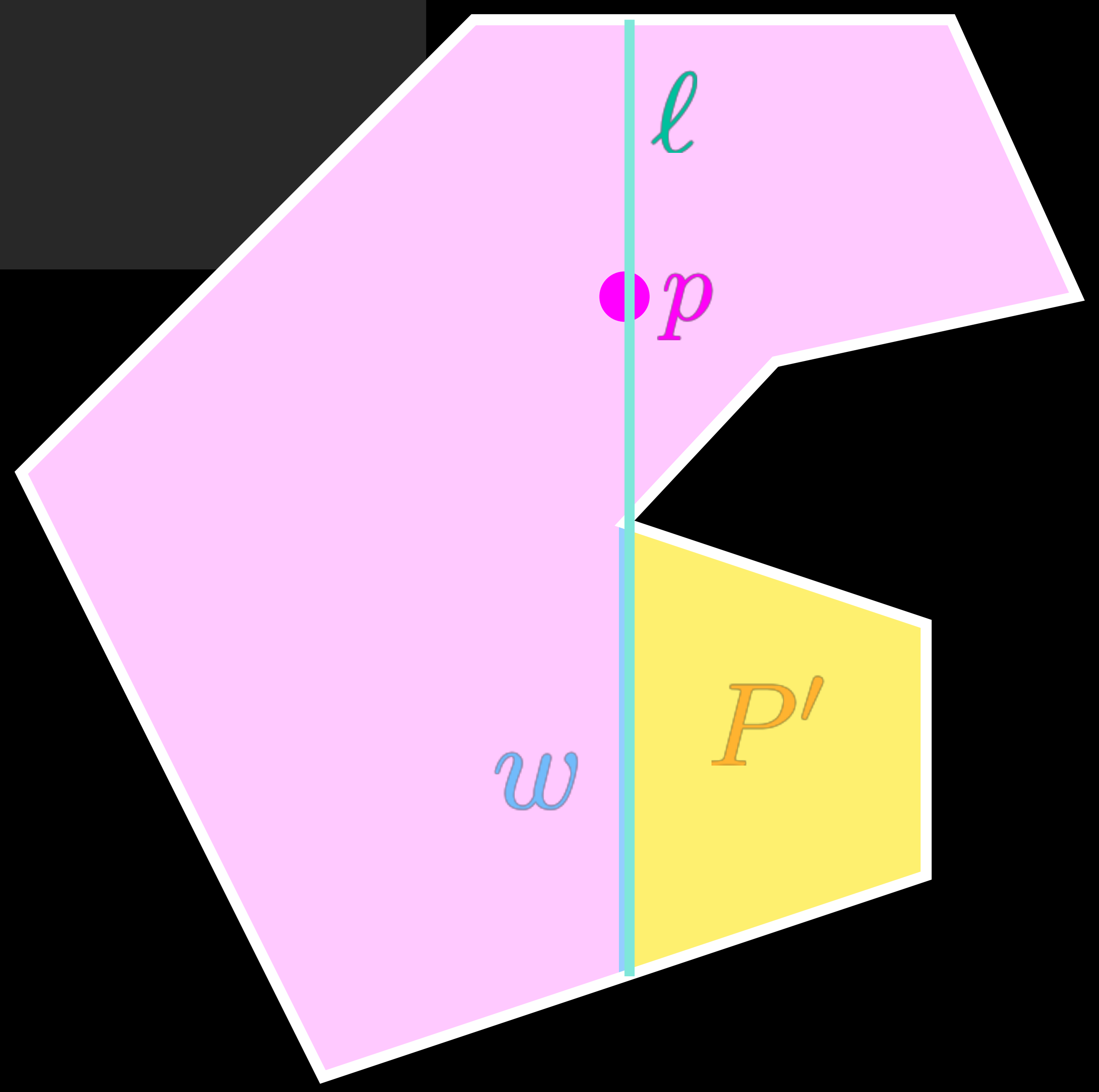


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- Splits $P$ into at least two subpolygons.

Subpolygon of $P$, no point visible from $p$

Line segment separating $P'$ and $P \setminus P'$

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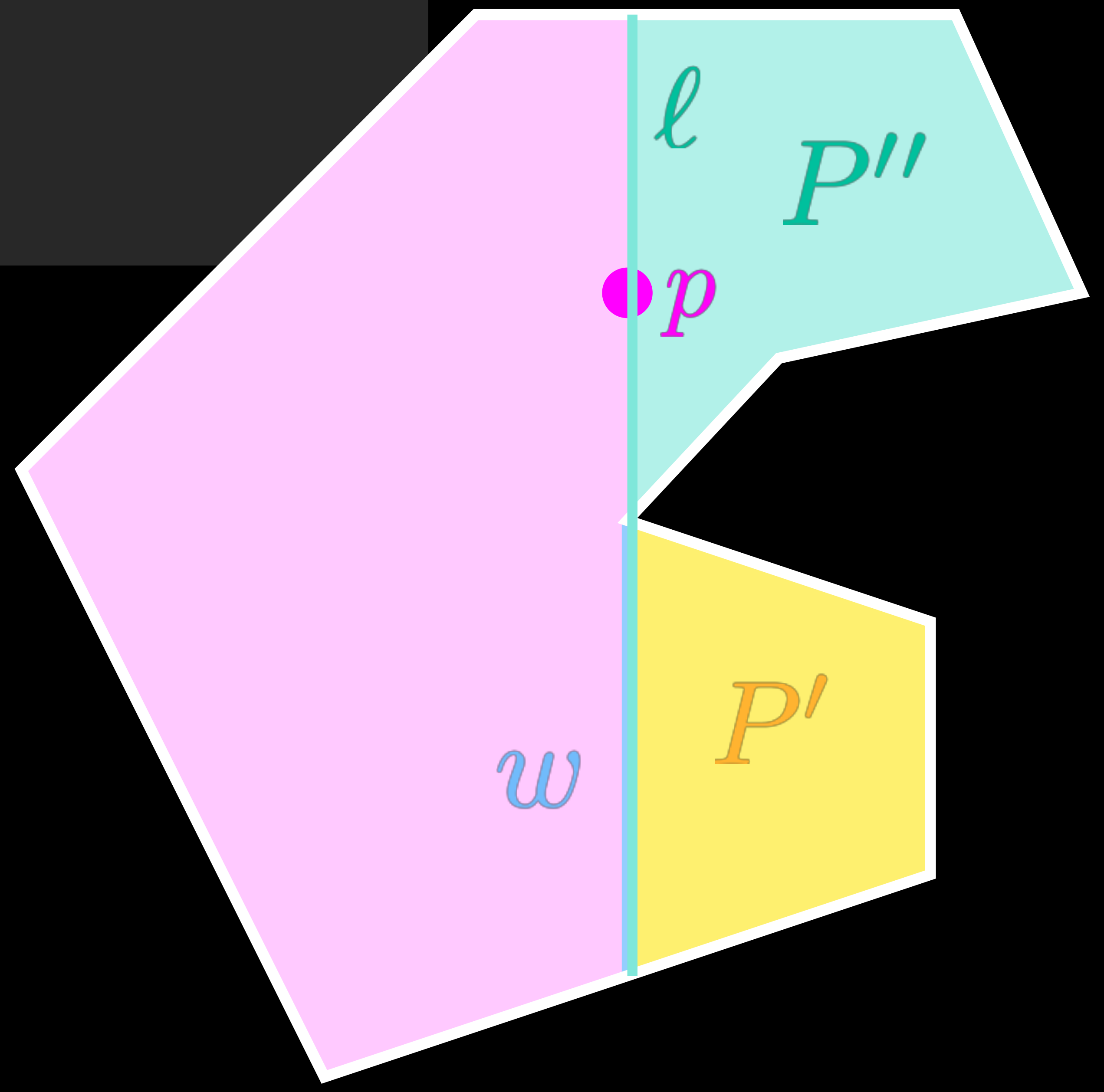


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Extend window $w$ of the pocket into a maximal line segment $\ell$

We know: $p \in \ell$ ➜ Segment $\ell \setminus w$ :

- Is polygonal edge with a convex endpoint not seen by $W_i$
- Splits $P$ into at least two subpolygons. At least one of those ($P''$) also right of $\ell$ ➜ $W_i$ cannot see any convex vertex in $P''$

Subpolygon of $P$, no point visible from $p$
Line segment separating $P'$ and $P \setminus P'$

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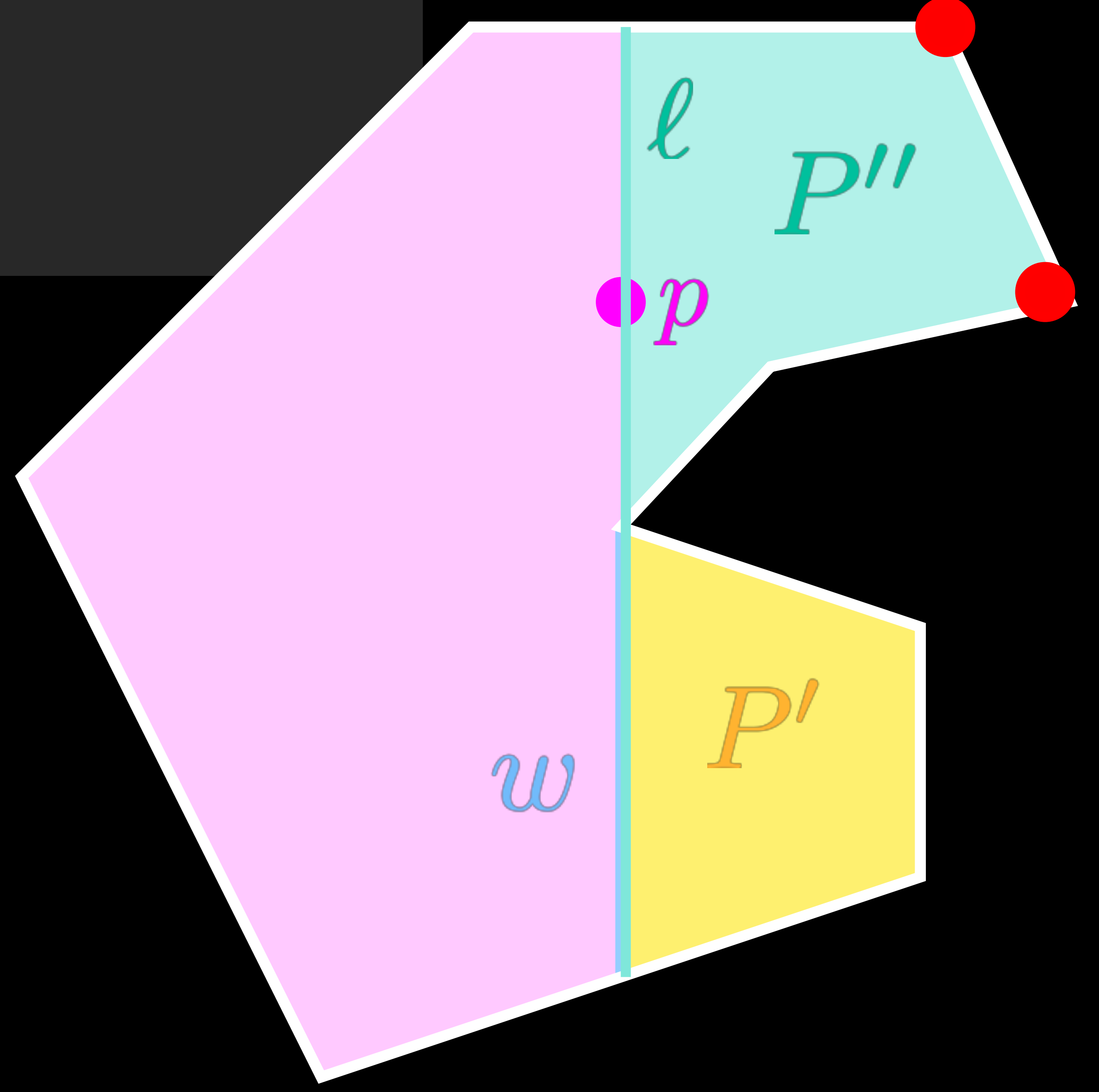


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Proof ctd: (1)-(3) $\Rightarrow$ $W_1$ and $W_2$ are segment watchman routes

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Proof ctd: (1)-(3) $\Rightarrow$ $W_1$ and $W_2$ are segment watchman routes

Consider $p \in P$

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Both $W_1$ and $W_2$ are watchman routes

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Proof ctd: (1)-(3) $\Rightarrow$ $W_1$ and $W_2$ are segment watchman routes

Consider $p \in P$

Both $W_1$ and $W_2$ are watchman routes

➜ At least one point on each route sees $p$

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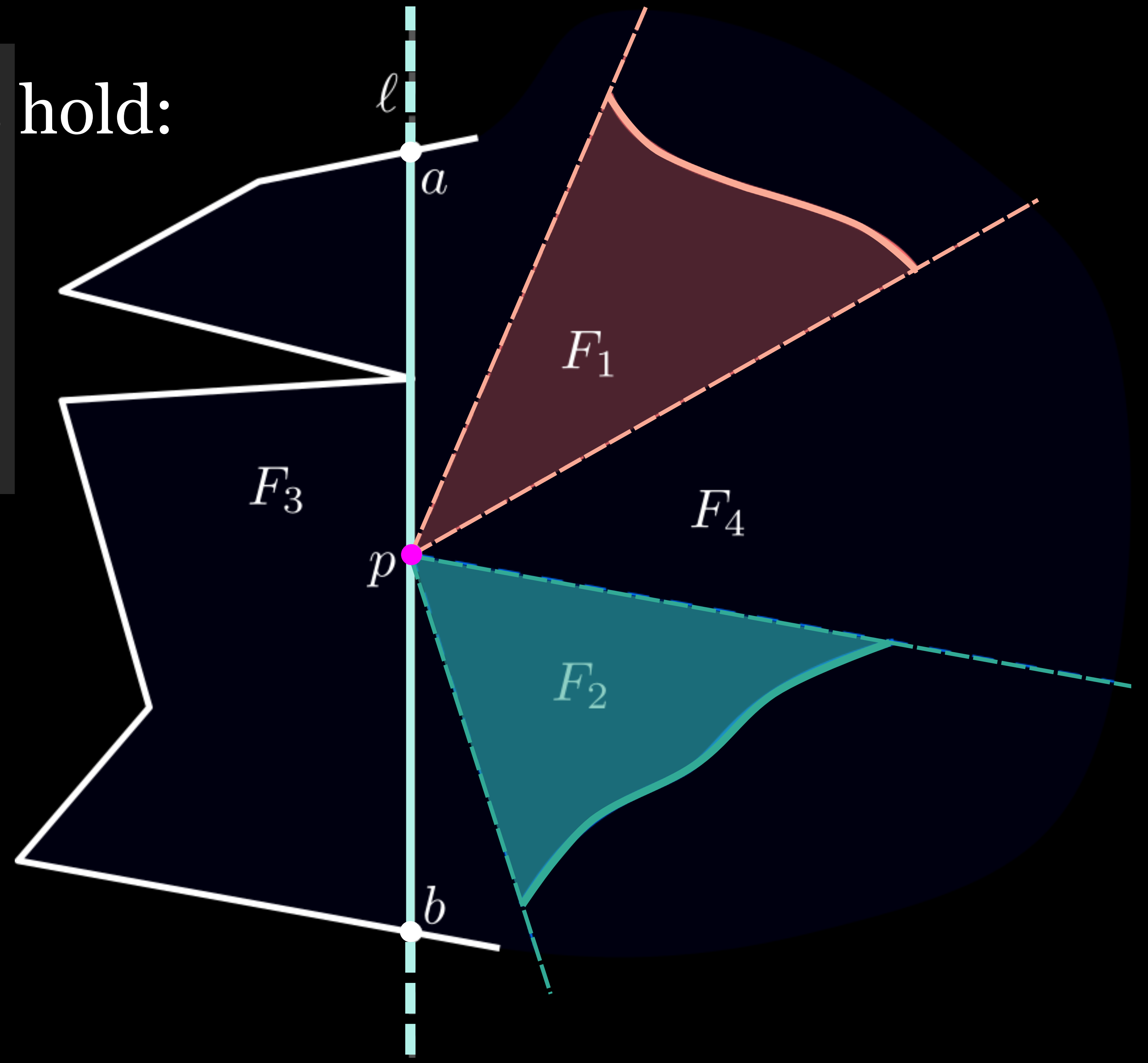


Proof ctd: (1)-(3) $\Rightarrow$ $W_1$ and $W_2$ are segment watchman routes

Consider $p \in P$

Both $W_1$ and $W_2$ are watchman routes

➜ At least one point on each route sees $p$

Consider two maximal wedges defined by angles from which $p$ views $W_i$— $F_i$

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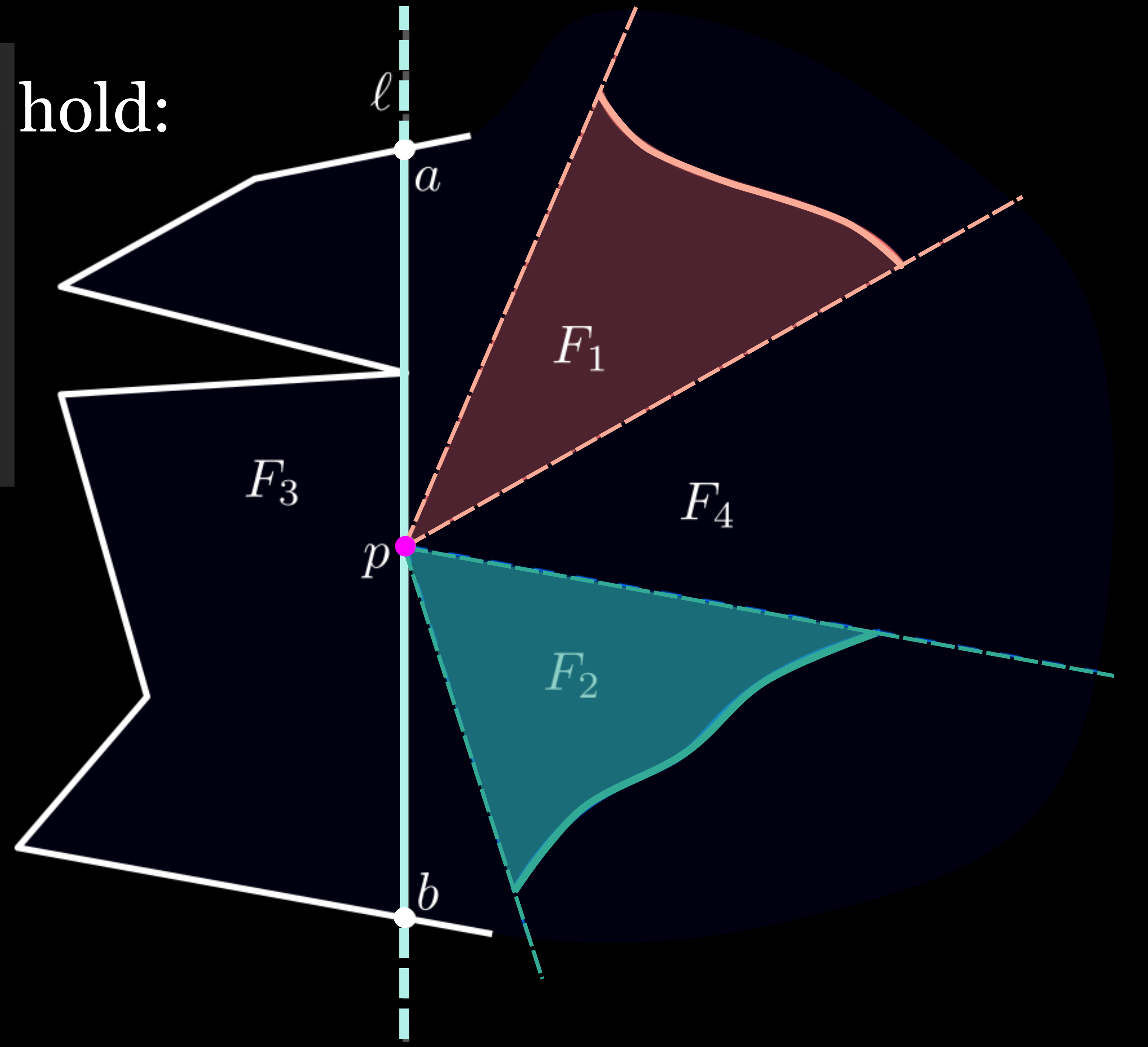


Proof ctd: (1)-(3) $\Rightarrow$ $W_1$ and $W_2$ are segment watchman routes

Consider $p \in P$

Both $W_1$ and $W_2$ are watchman routes

➜ At least one point on each route sees $p$

Consider two maximal wedges defined by angles from which $p$ views $W_i$— $F_i$

Each of $F_1$ and $F_2$ covers either 360° or less than 180° ($p$ within RCH and routes relatively convex):

*For any two points inside the region enclosed by the route, their shortest path is also contained within the enclosed region

# Sufficient Conditions: The "Conditions Lemma"

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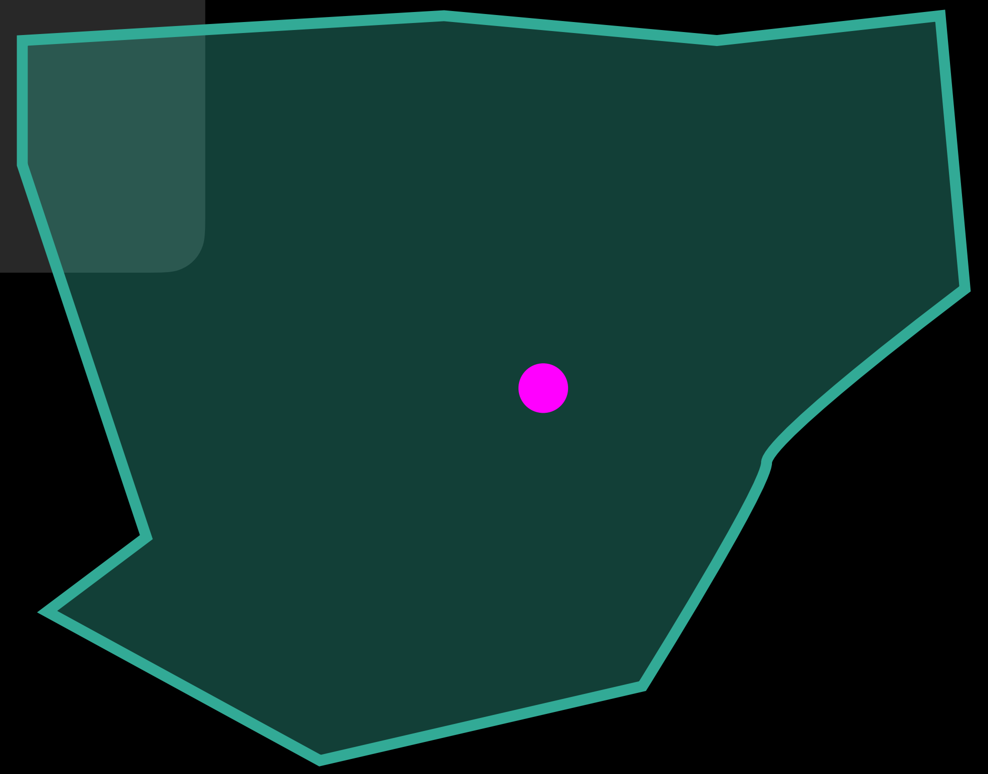

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Consider $p \in P$

Both $W_1$ and $W_2$ are watchman routes

➜ At least one point on each route sees $p$

Consider two maximal wedges defined by angles from which $p$ views $W_i$— $F_i$

Each of $F_1$ and $F_2$ covers either 360° or less than 180° ($p$ within RCH and routes relatively convex):

- One ($F_2$) covers 360°: let $w_1 \in W_1$ be a point that sees $p$

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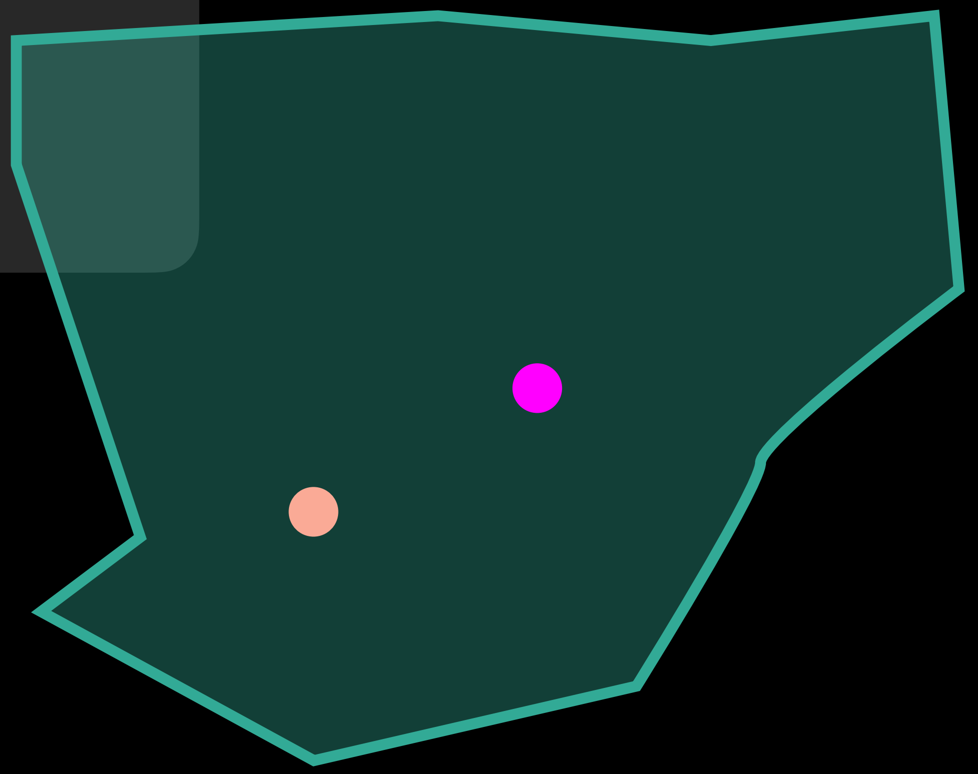

Proof ctd: (1)-(3) ⇒ $W_1$ and $W_2$ are segment watchman routes

Consider $p \in P$

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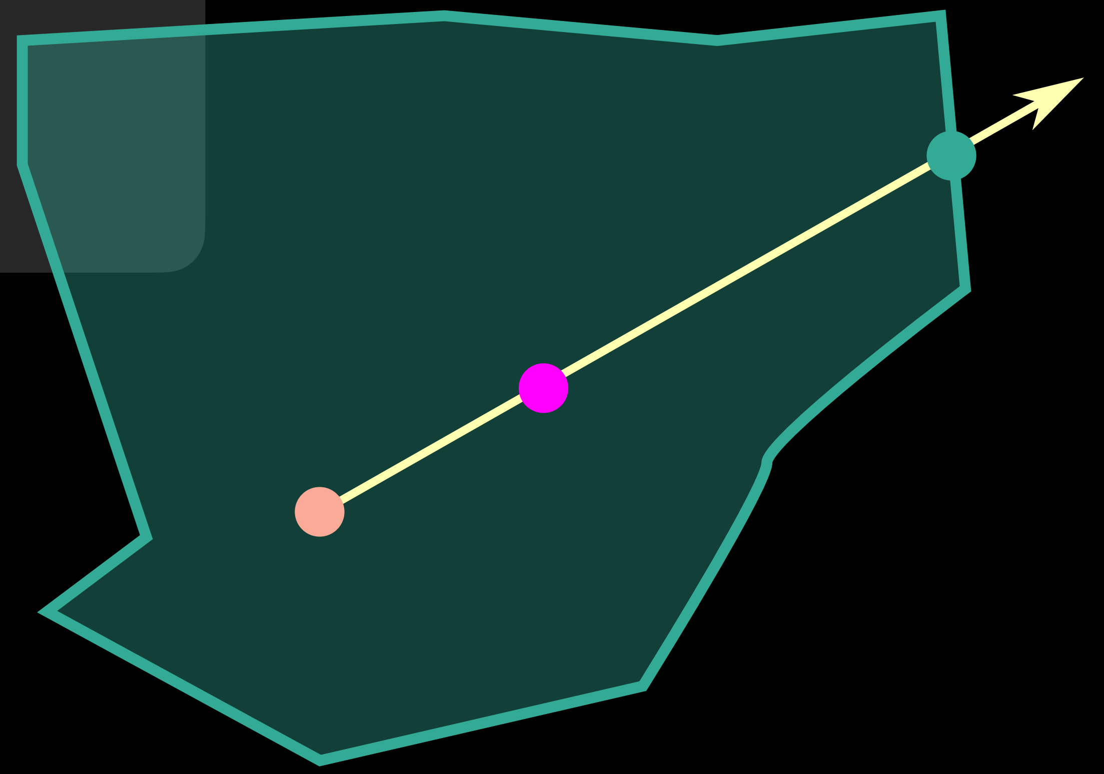

Proof ctd: (1)-(3) ⇒ $W_1$ and $W_2$ are segment watchman routes

Consider $p \in P$

Both $W_1$ and $W_2$ are watchman routes

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Each of $F_1$ and $F_2$ covers either 360° or less than 180° ($p$ within RCH and routes relatively convex):

- One ($F_2$) covers 360°: let $w_1 \in W_1$ be a point that sees $p$

➡ Ray from $w_1$ in direction of $p$ intersects $W_2$ at point $w_2$ that sees $p$

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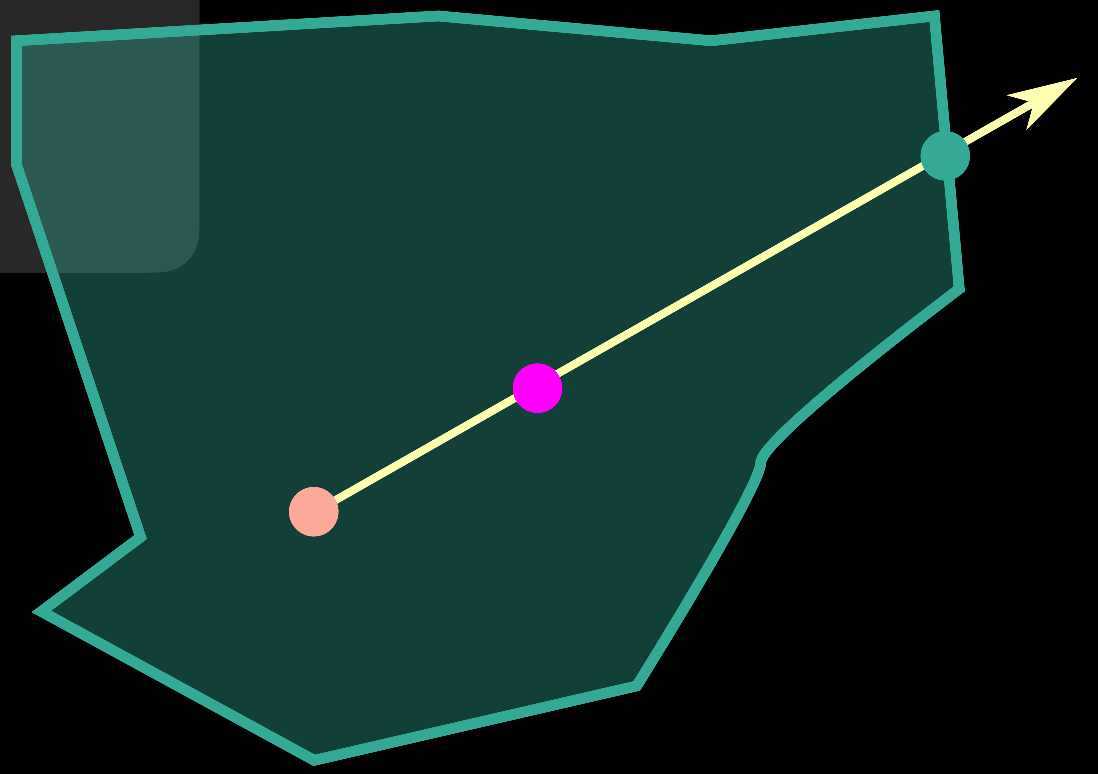

Proof ctd: (1)-(3) $\Rightarrow$ $W_1$ and $W_2$ are segment watchman routes

Consider $p \in P$

Both $W_1$ and $W_2$ are watchman routes

➜ At least one point on each route sees $p$

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- One ($F_2$) covers 360°: let $w_1 \in W_1$ be a point that sees $p$

➡ Ray from $w_1$ in direction of $p$ intersects $W_2$ at point $w_2$ that sees $p$

➡ $p$ is segment guarded by $\overline{w_1 w_2}$

*For any two points inside the region enclosed by the route, their shortest path is also contained within the enclosed region

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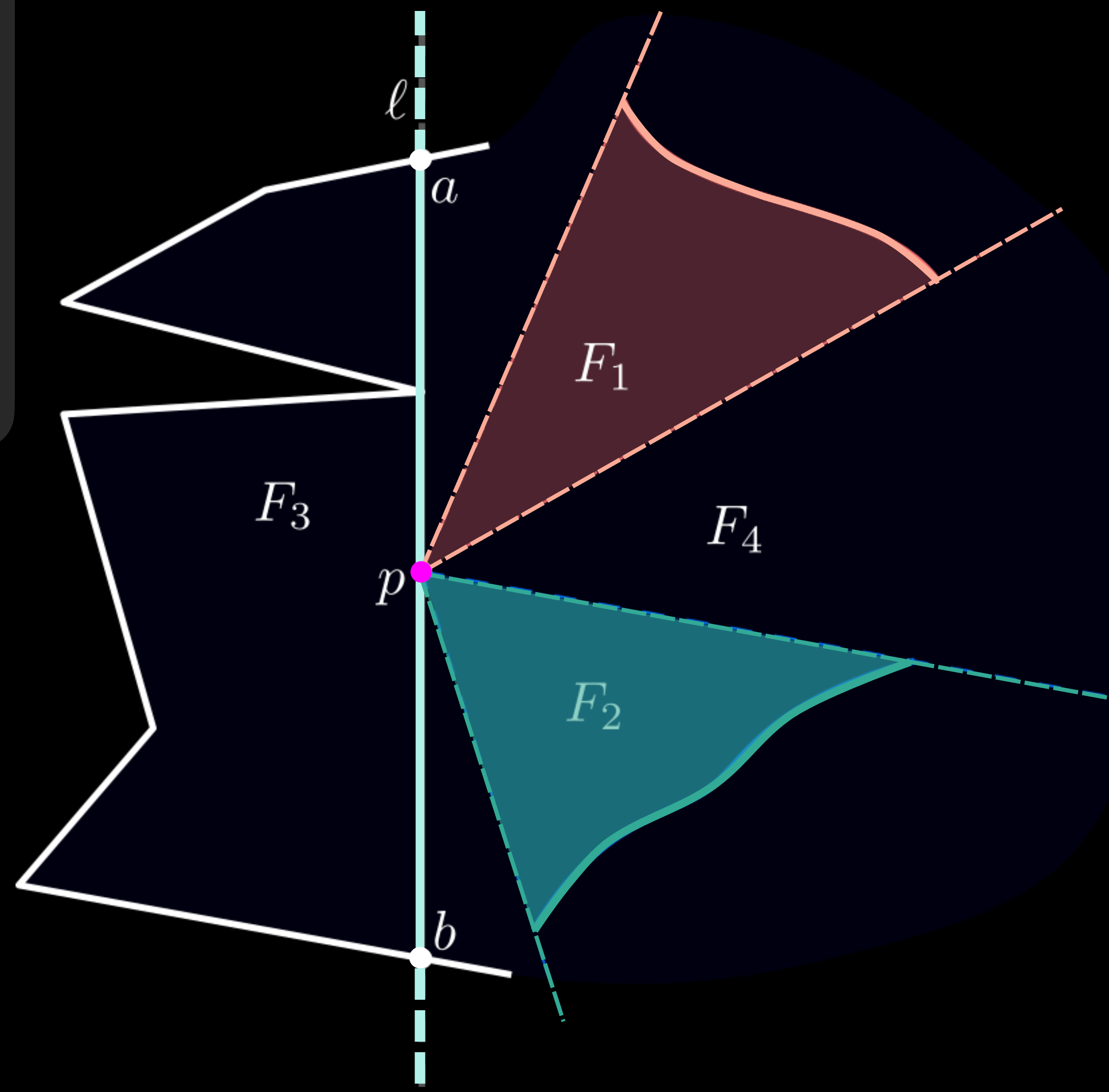


Proof ctd: (1)-(3) ⇒ $W_1$ and $W_2$ are segment watchman routes

- Neither $F_1$ nor $F_2$ covers 360°

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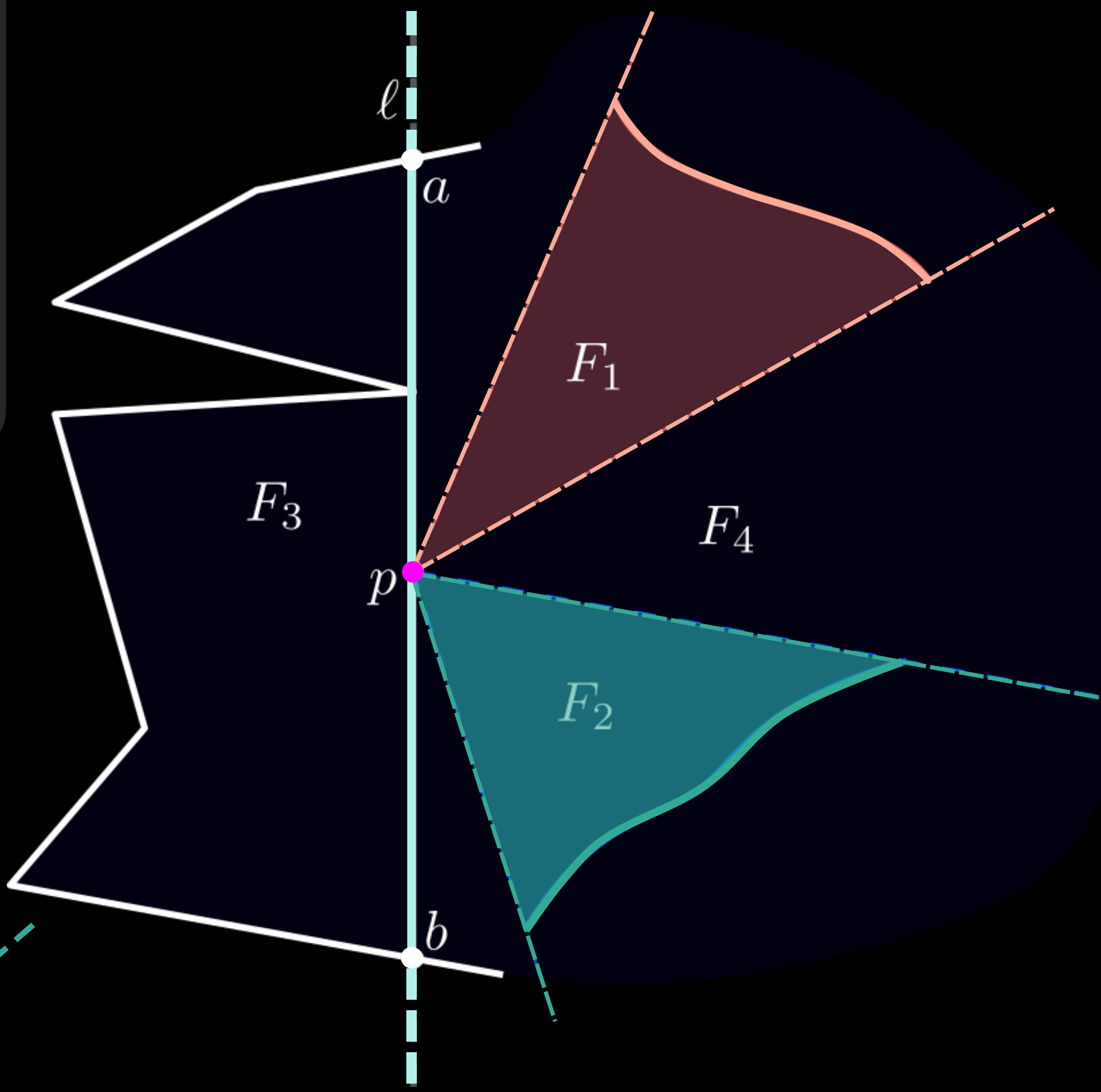


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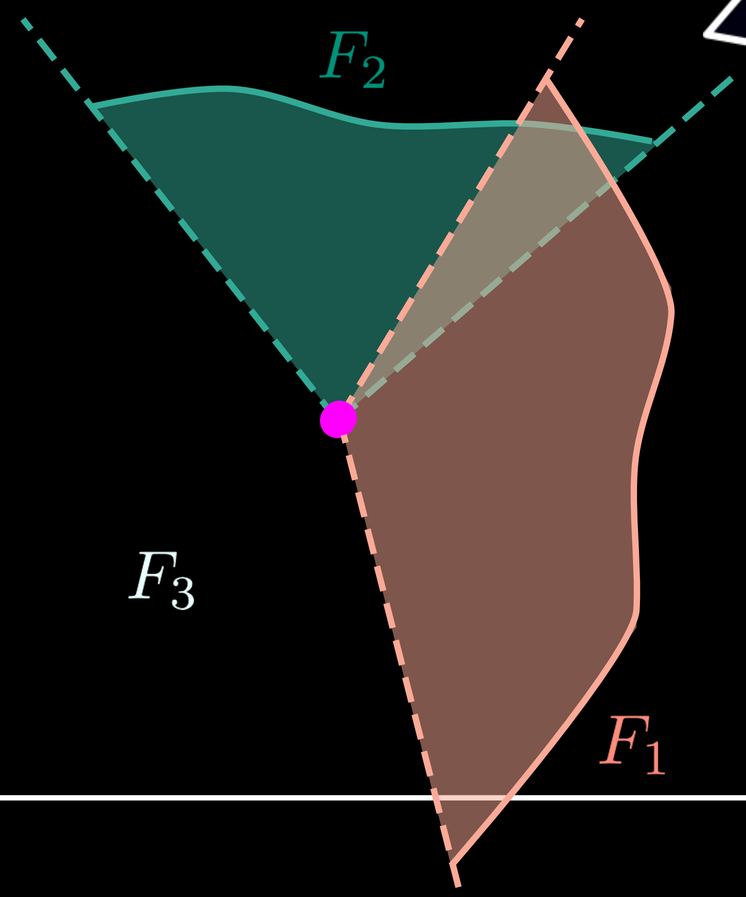

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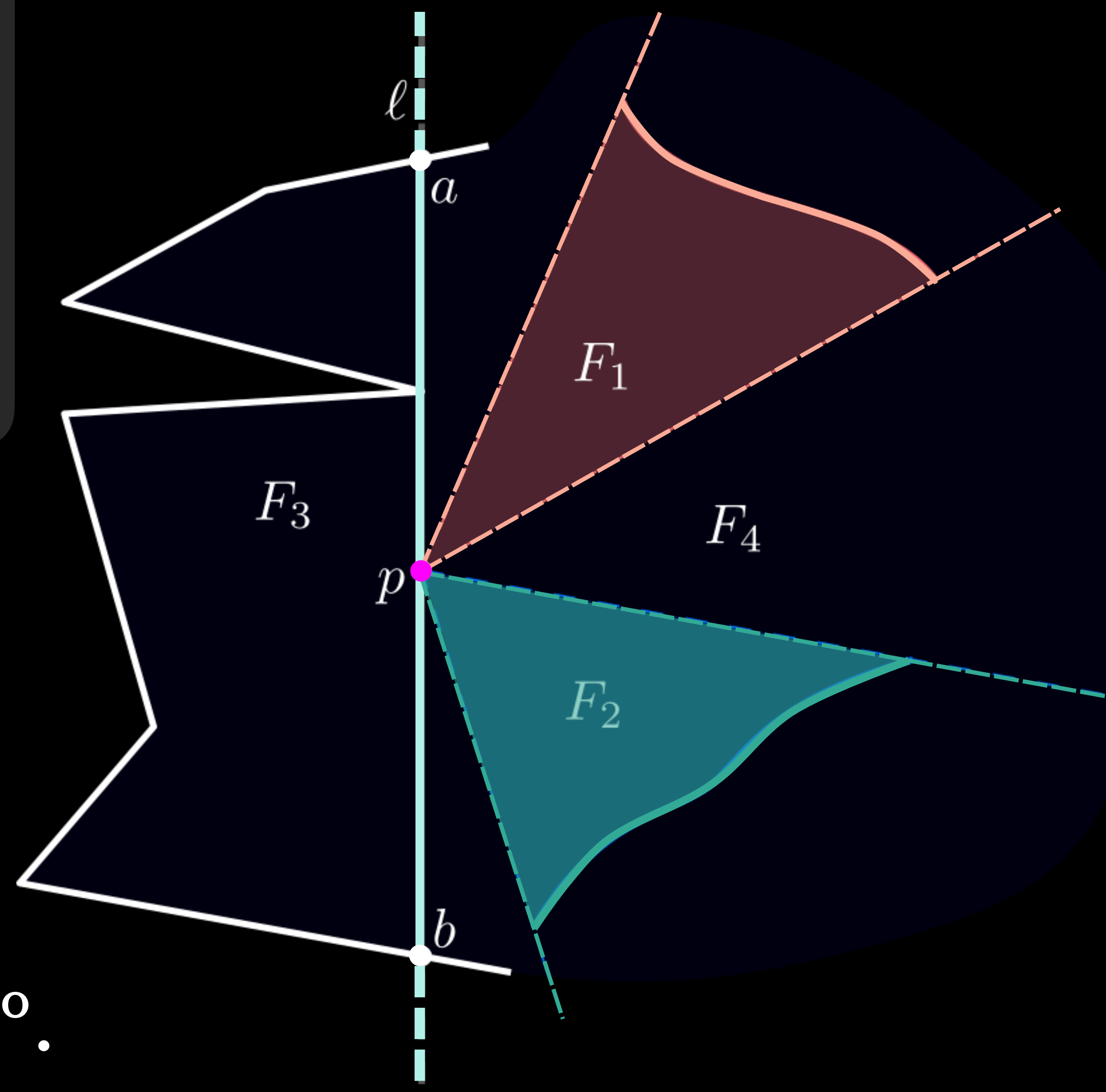


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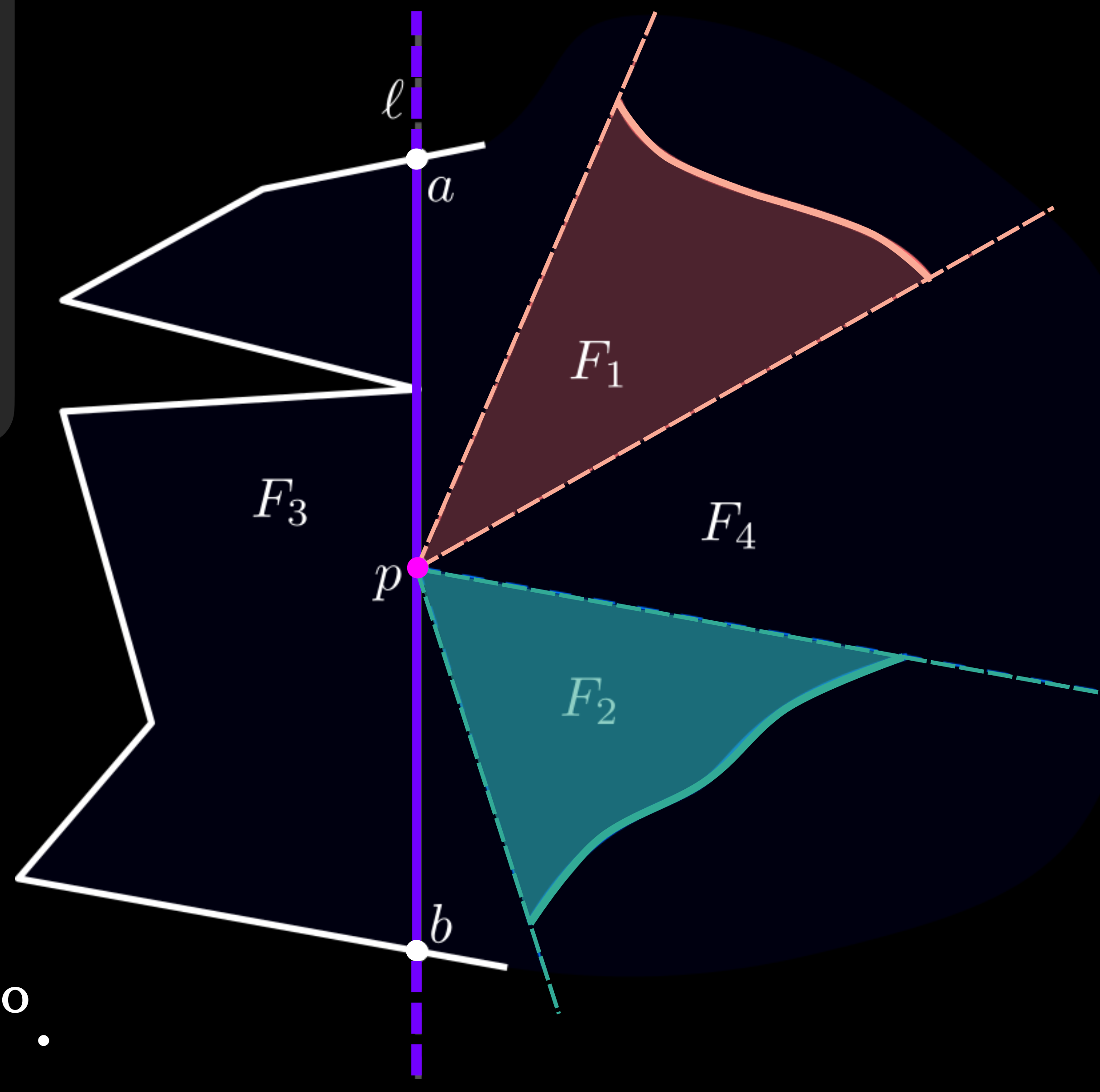


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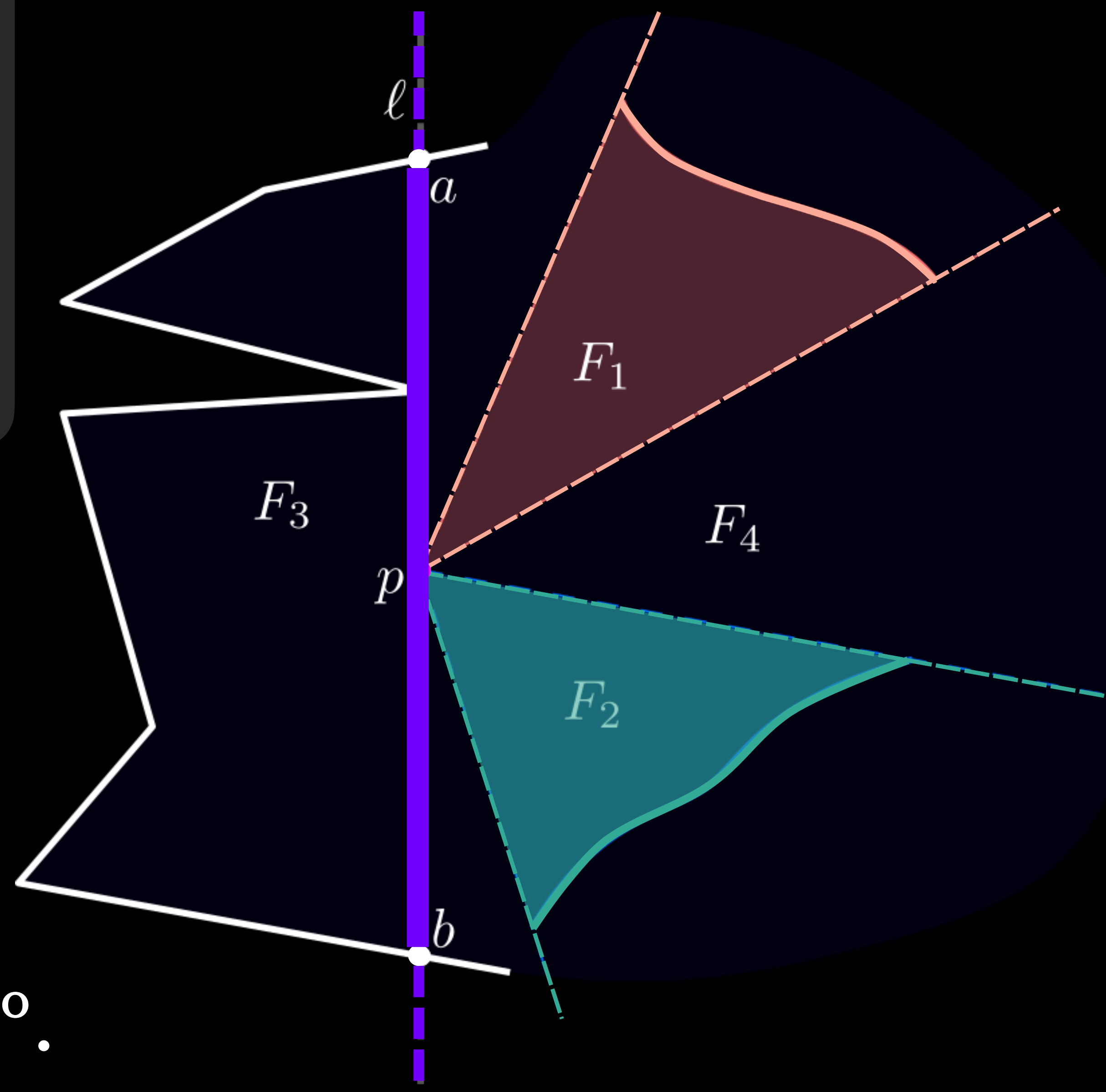


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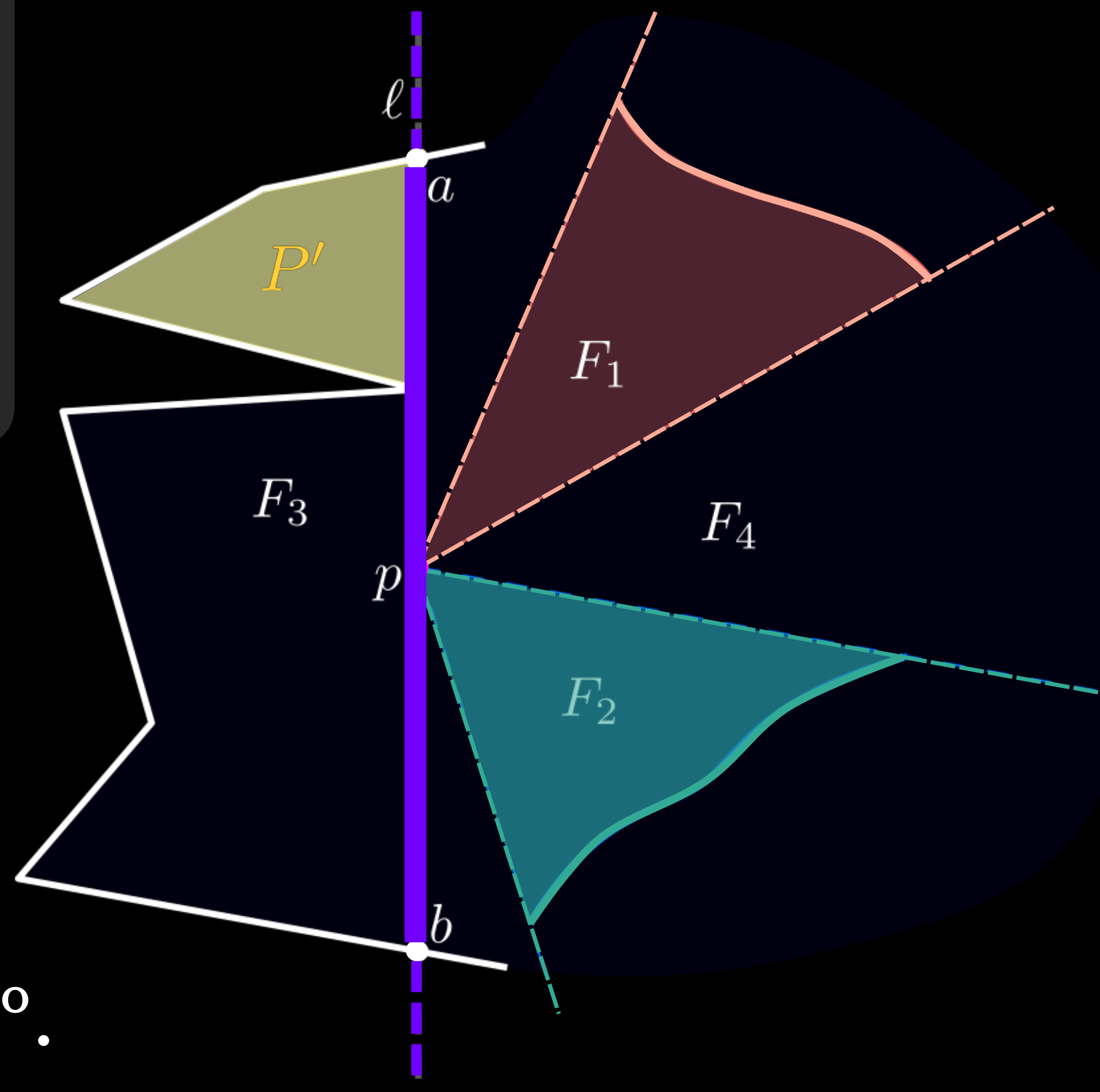


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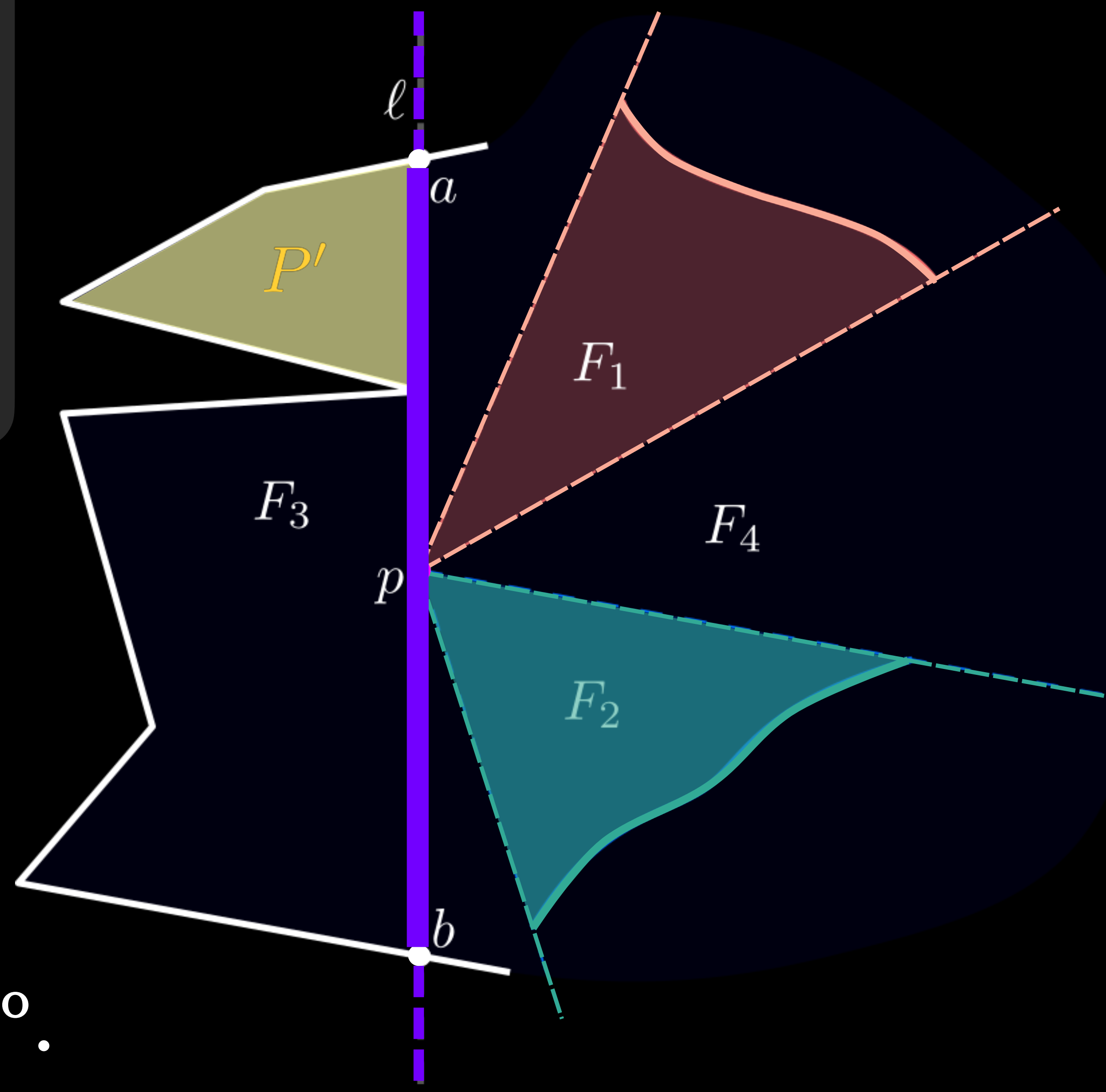


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$P'$ must contain a convex vertex $v$, but no points of $W_1$ *and* $W_2$ in $P'$ ↯

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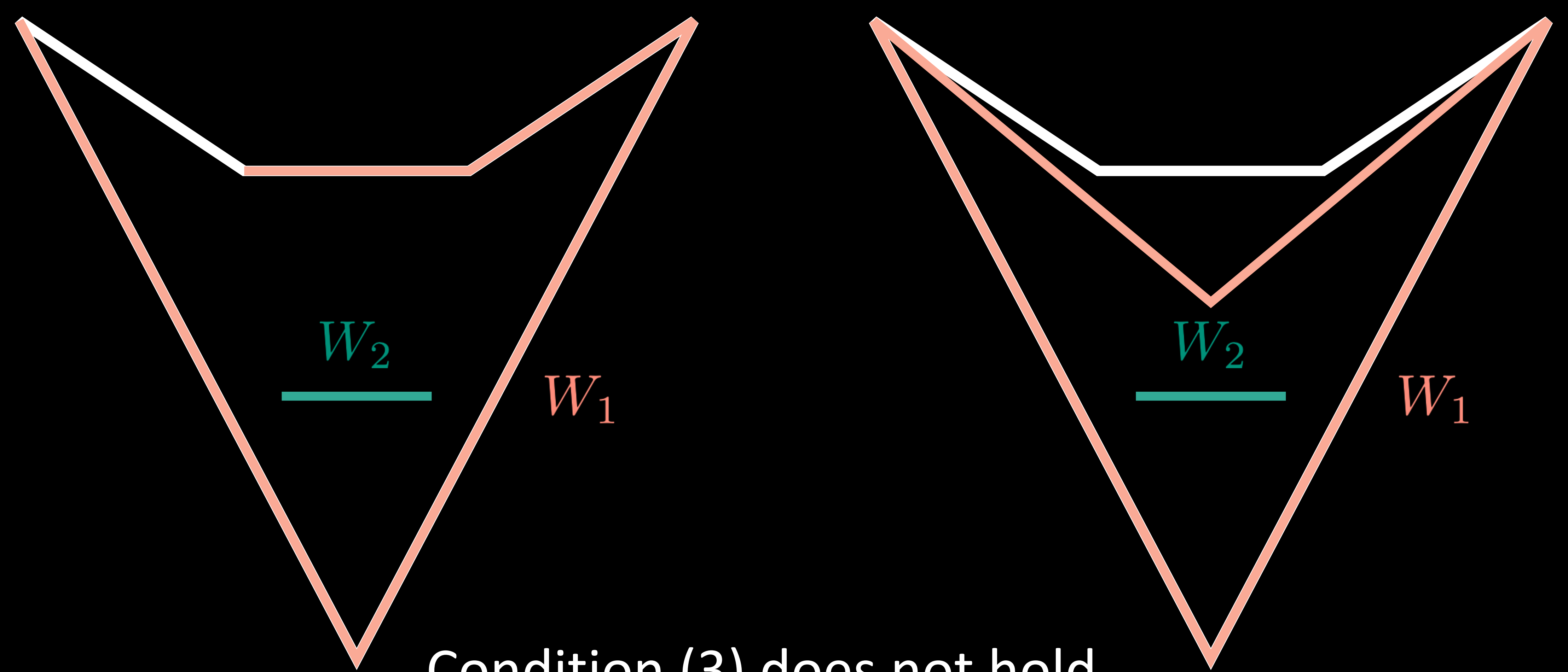
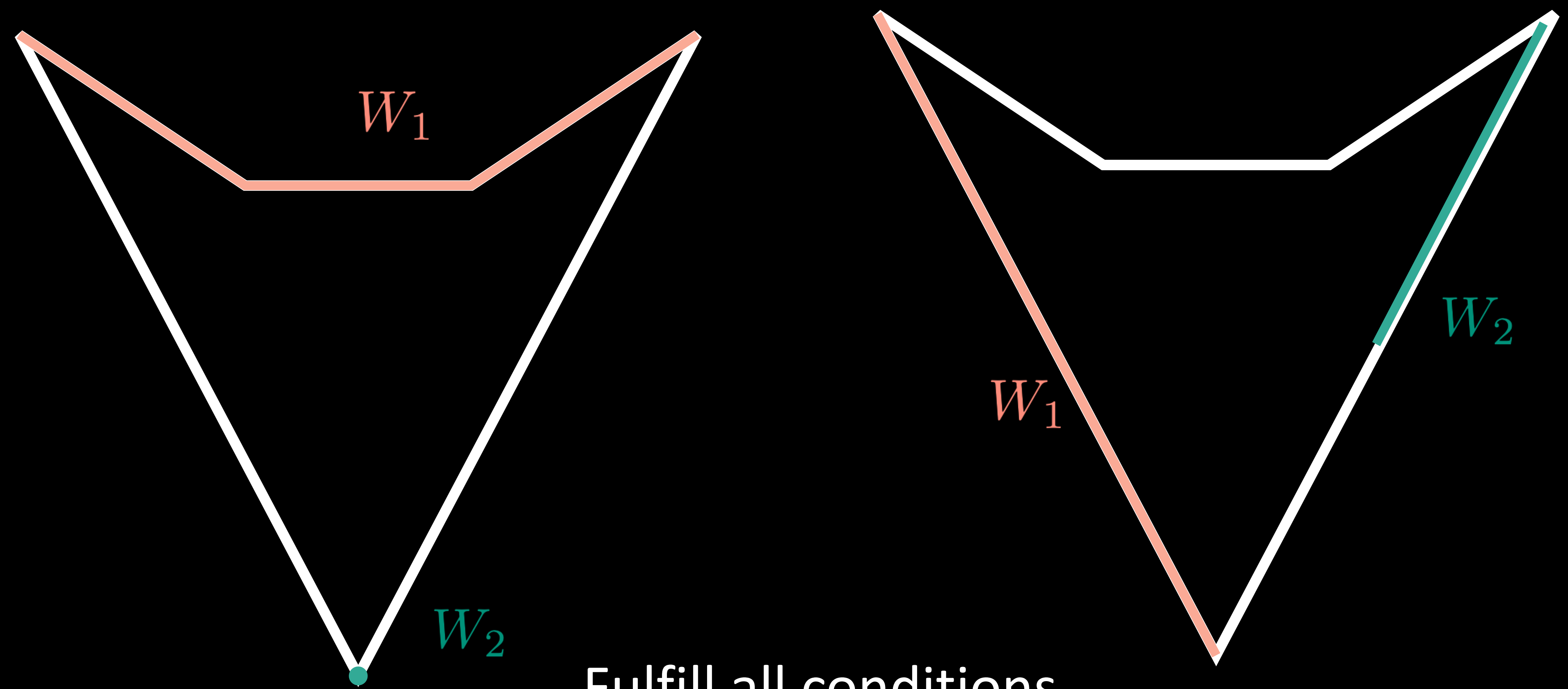



Condition (3) does not hold




Fulfill all conditions

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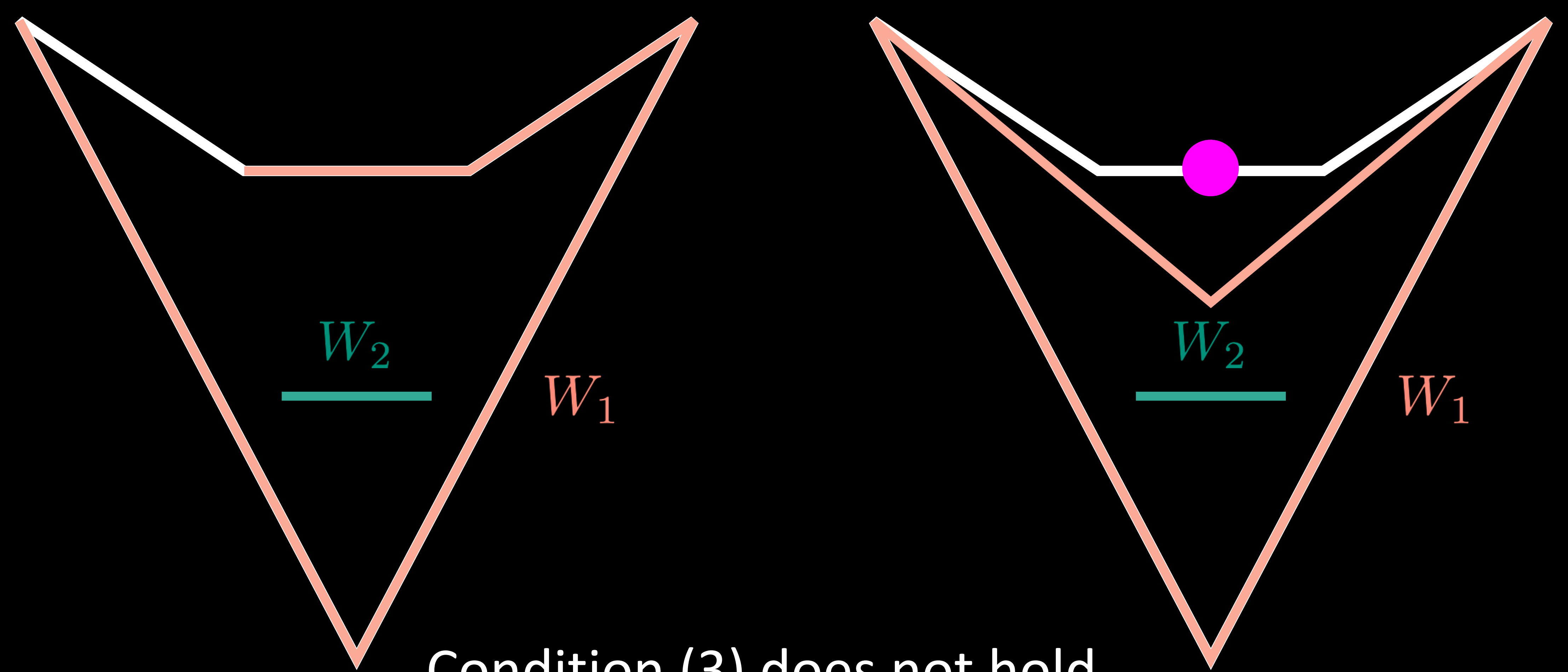


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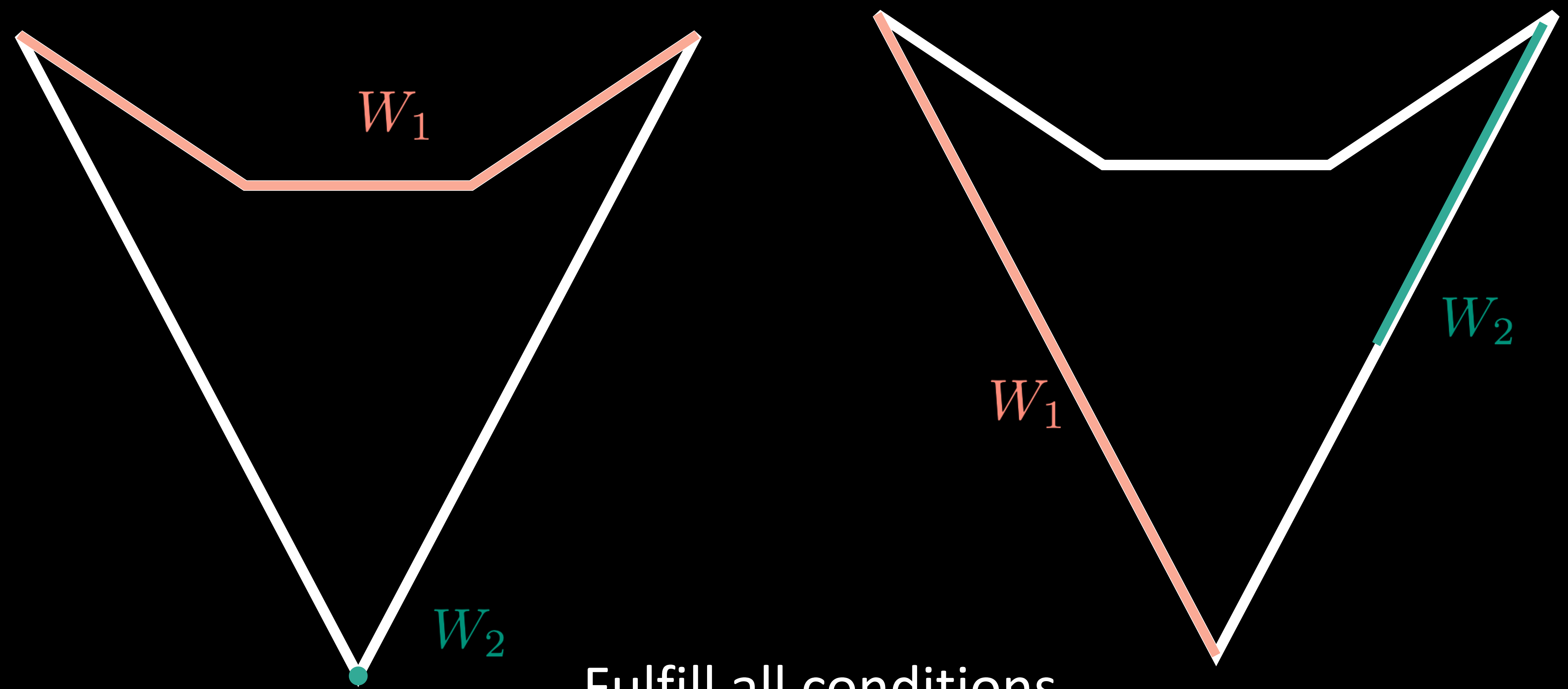


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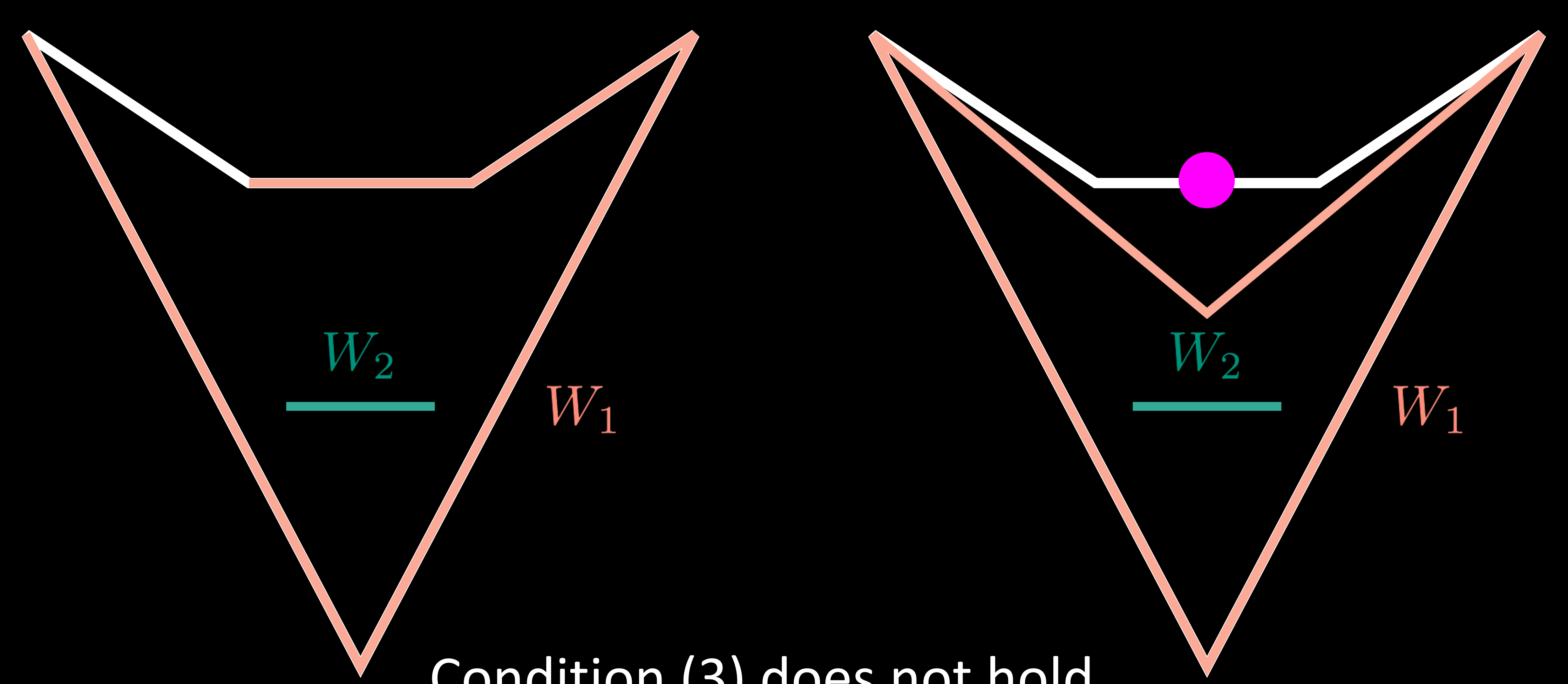
Two routes $W_1$ and $W_2$ are **optimal** segment watchman routes for $P$ **if and only if** conditions of the lemma hold.

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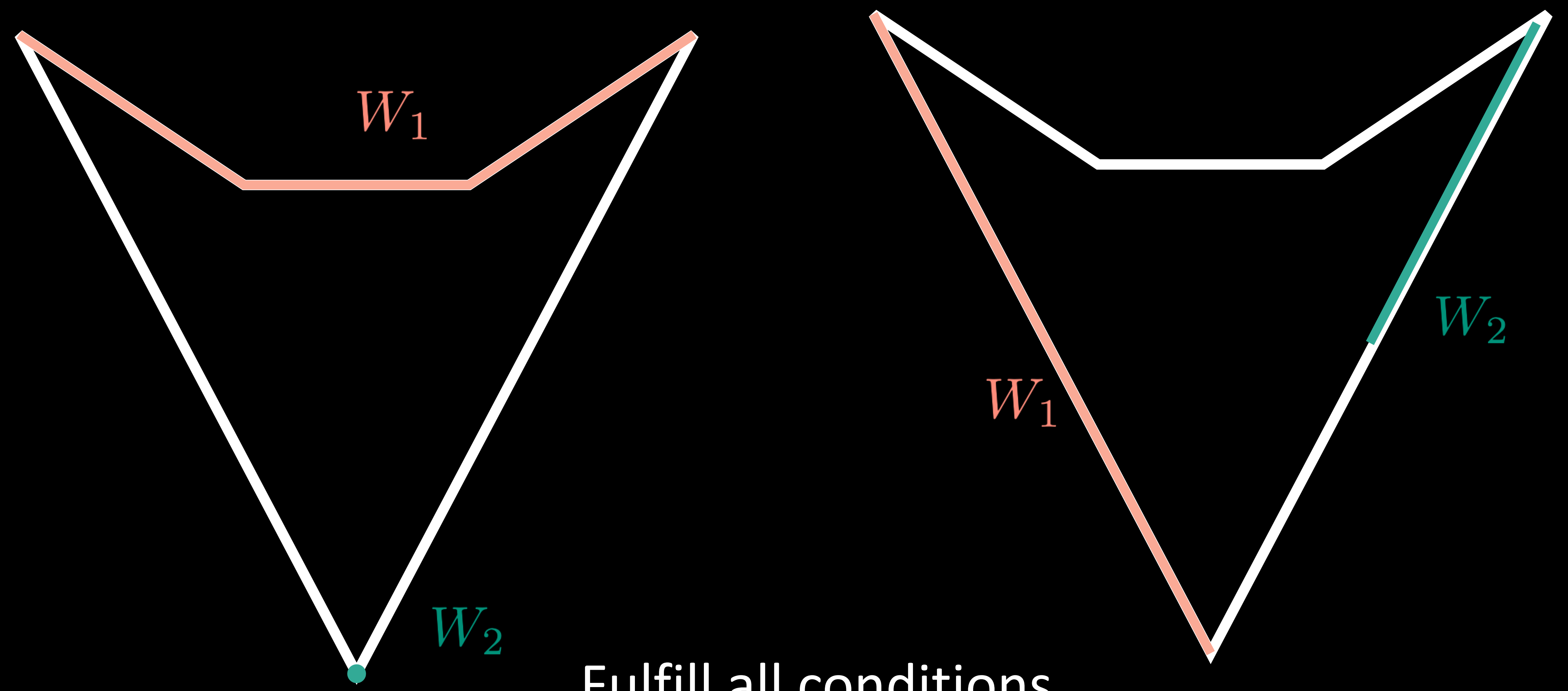


Fulfill all conditions

# Our Results

Min-max objective:

- NP-hard even for simple polygons
- Polynomial-time 2-approximation algorithm
- For larger $k$: $(k+1)$-approximation algorithm

Min-sum objective:

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# 2-Approximation Algorithm

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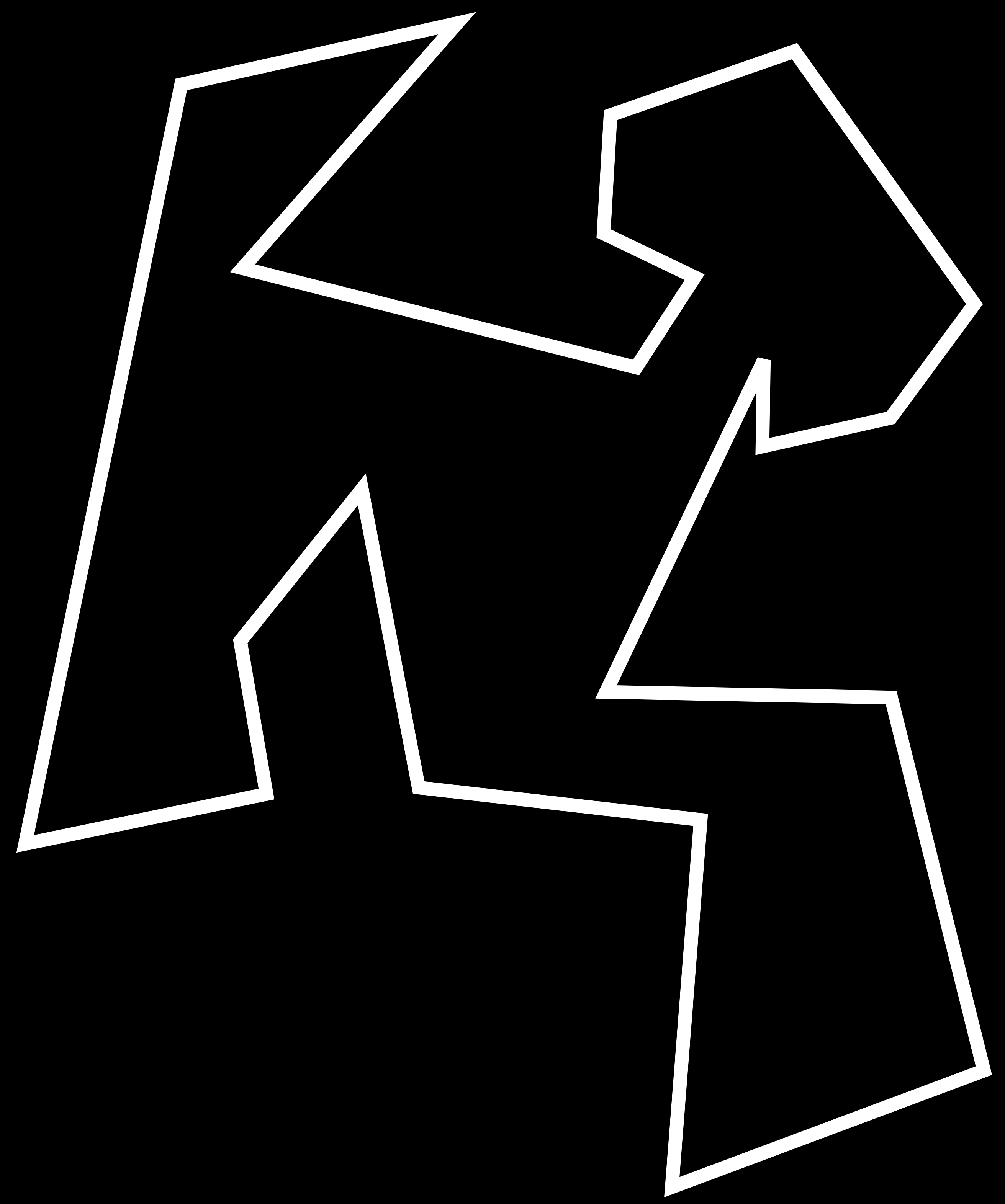

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Idea:

Each route:

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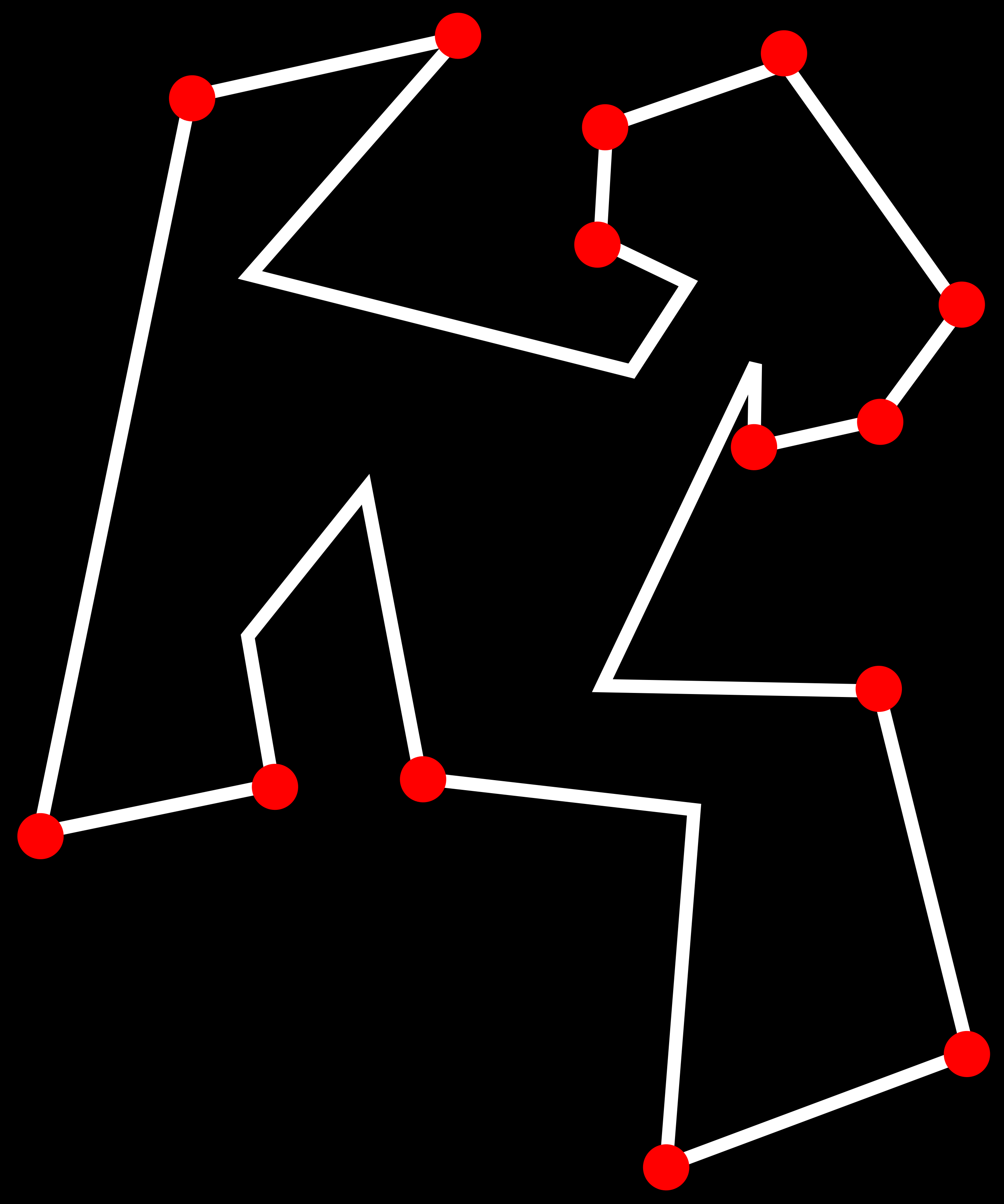

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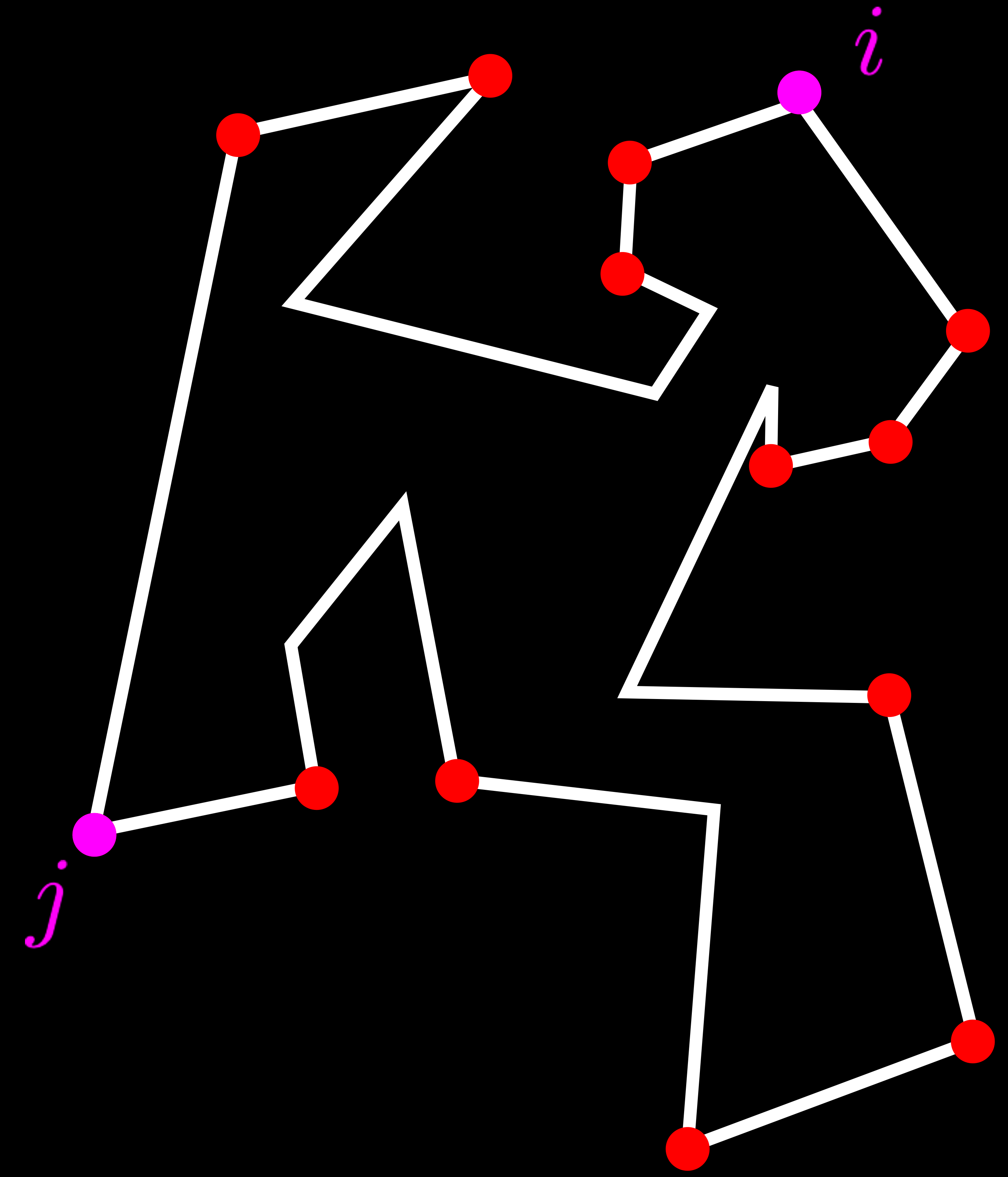

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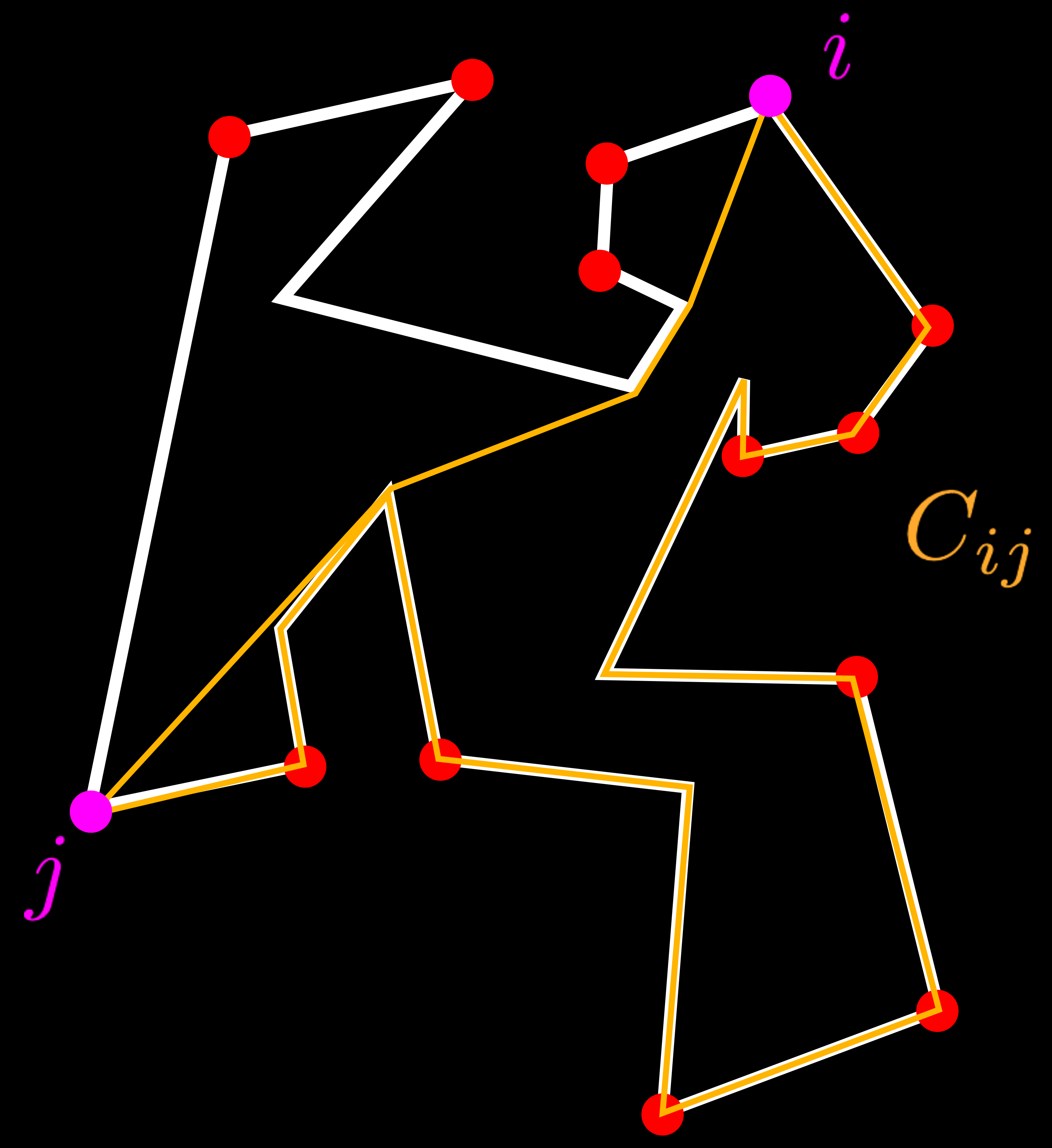

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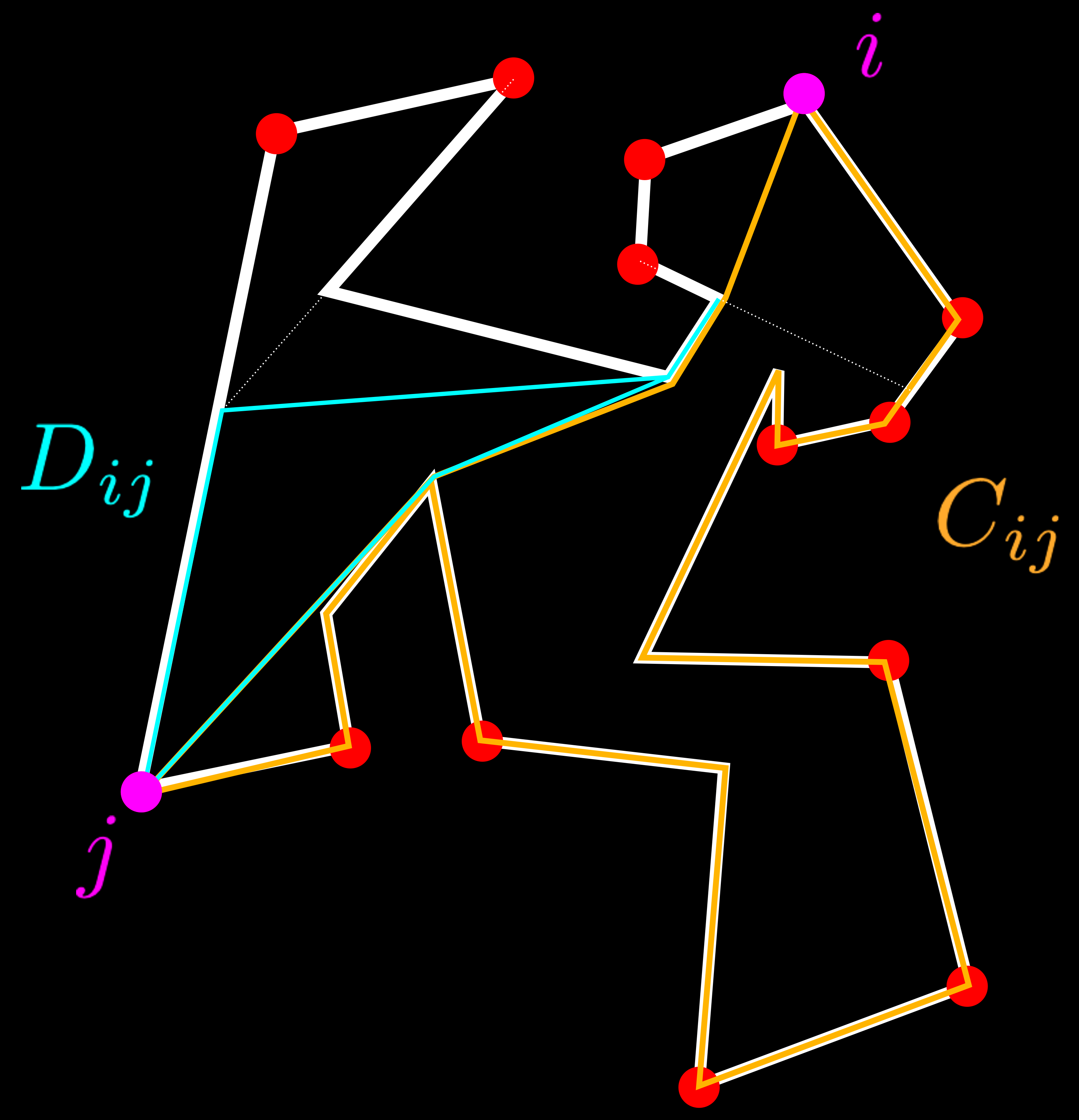

*The relative convex hull (RCH) of $C_{ij}$ and $D_{ij}$: the minimal set that contains $C_{ij}$ and $D_{ij}$ and is closed under taking shortest paths

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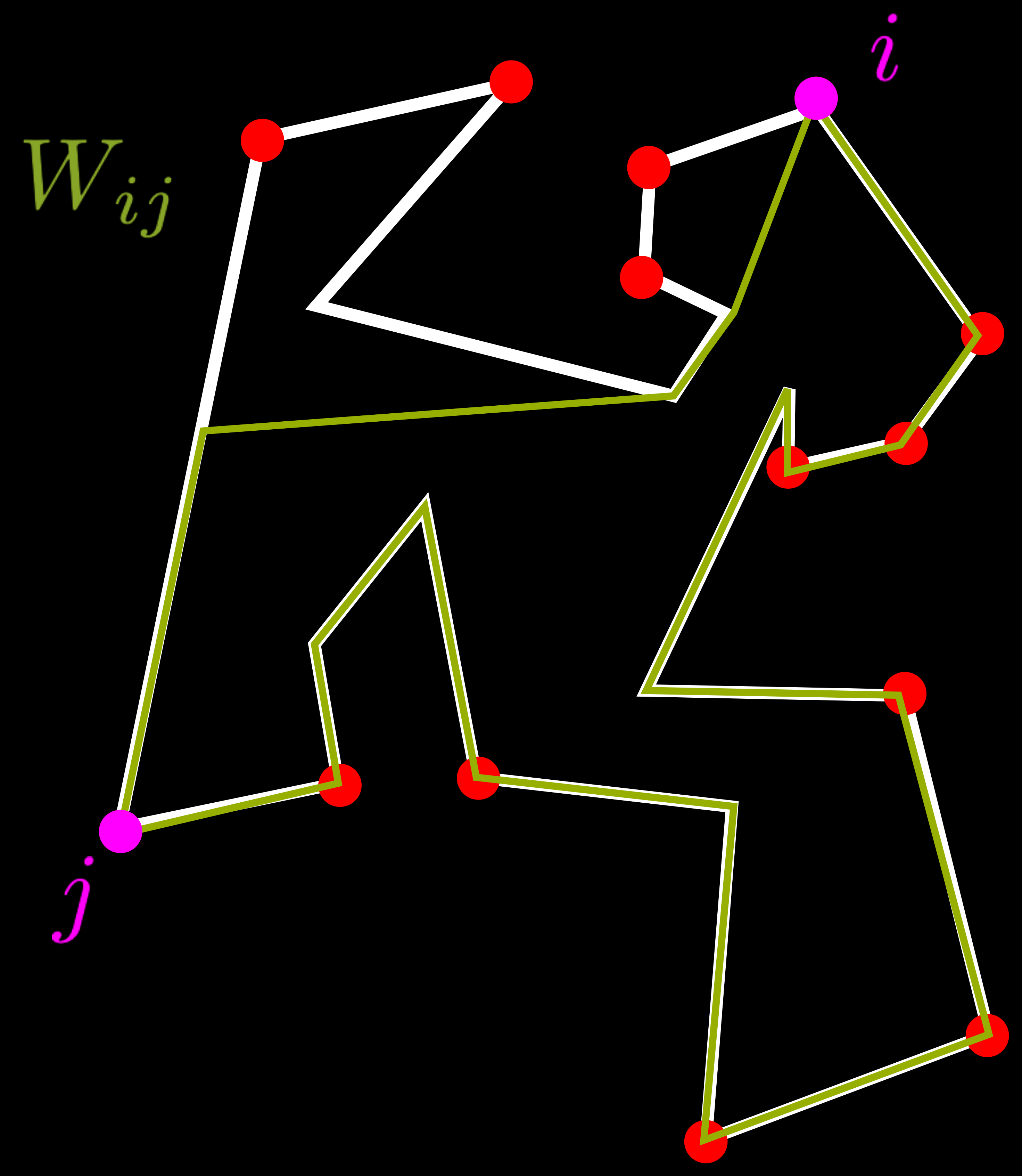

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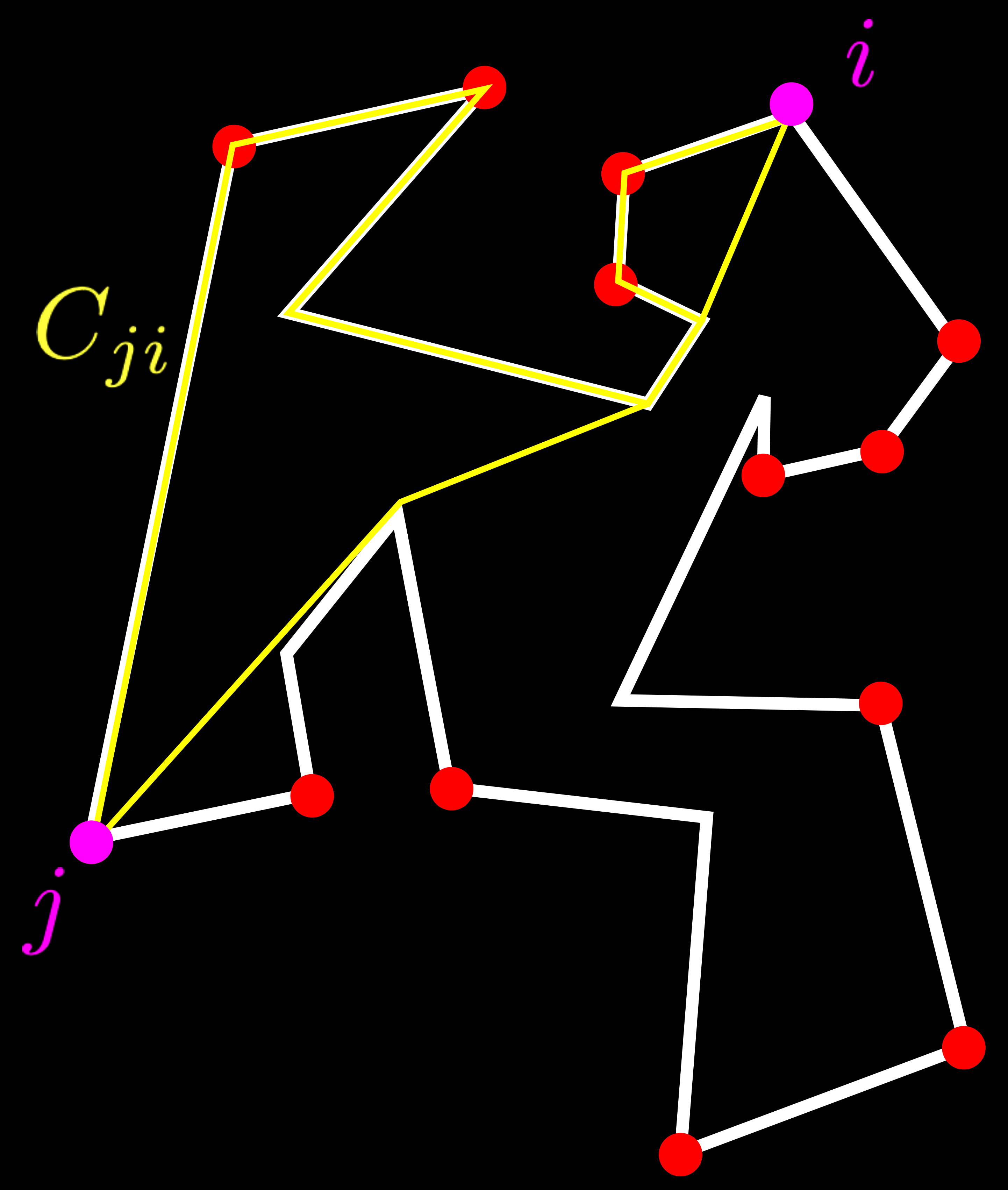

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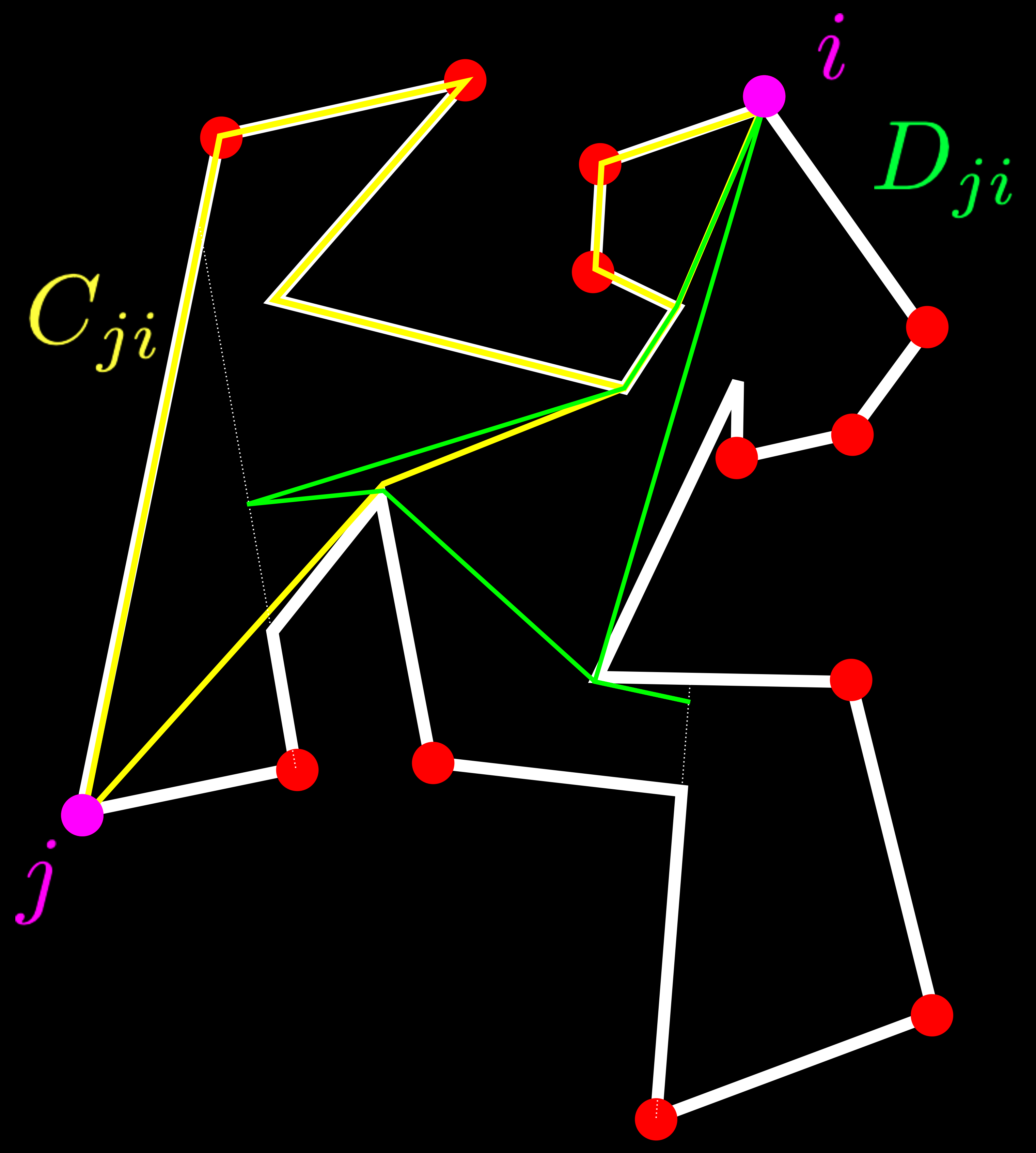

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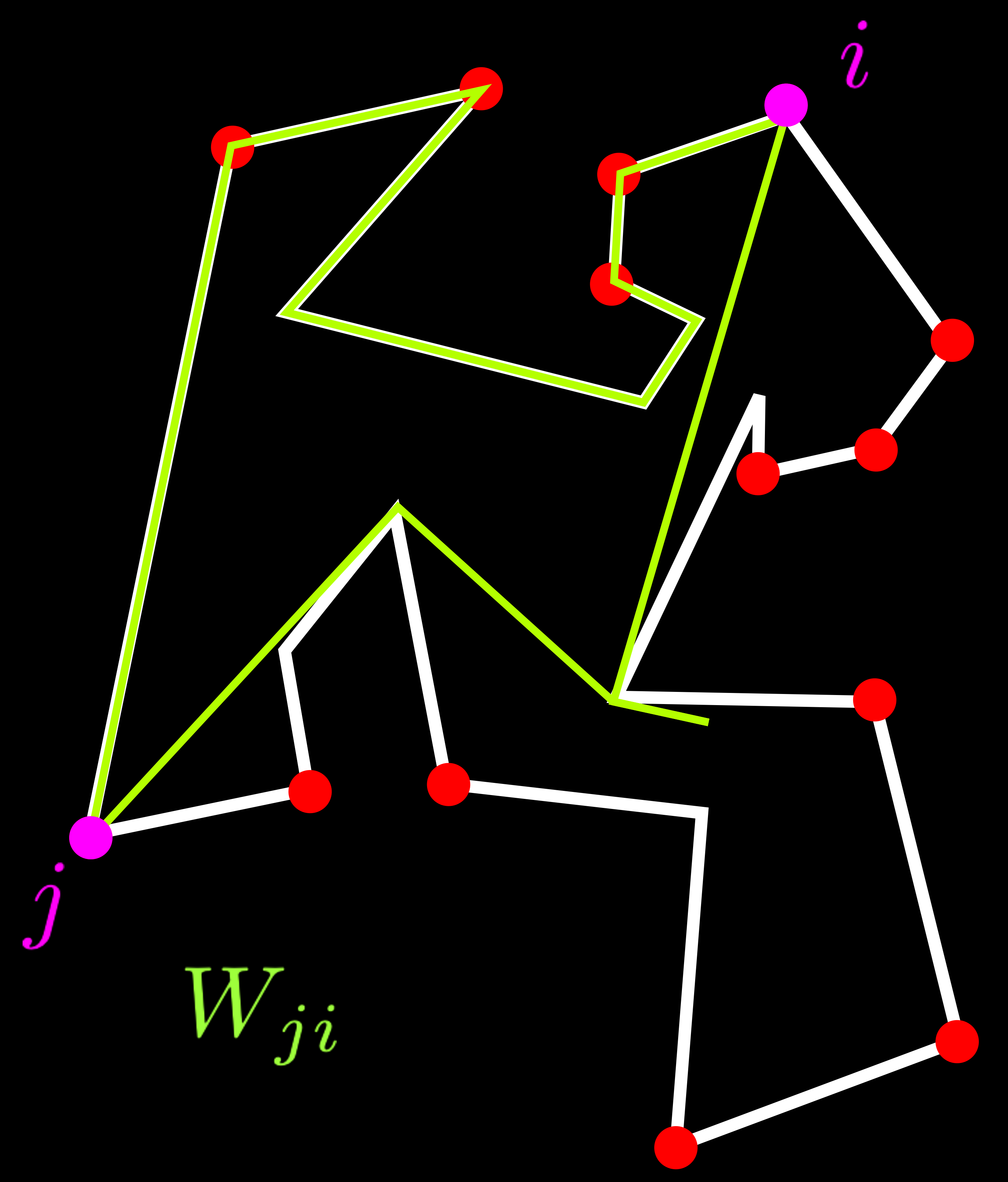

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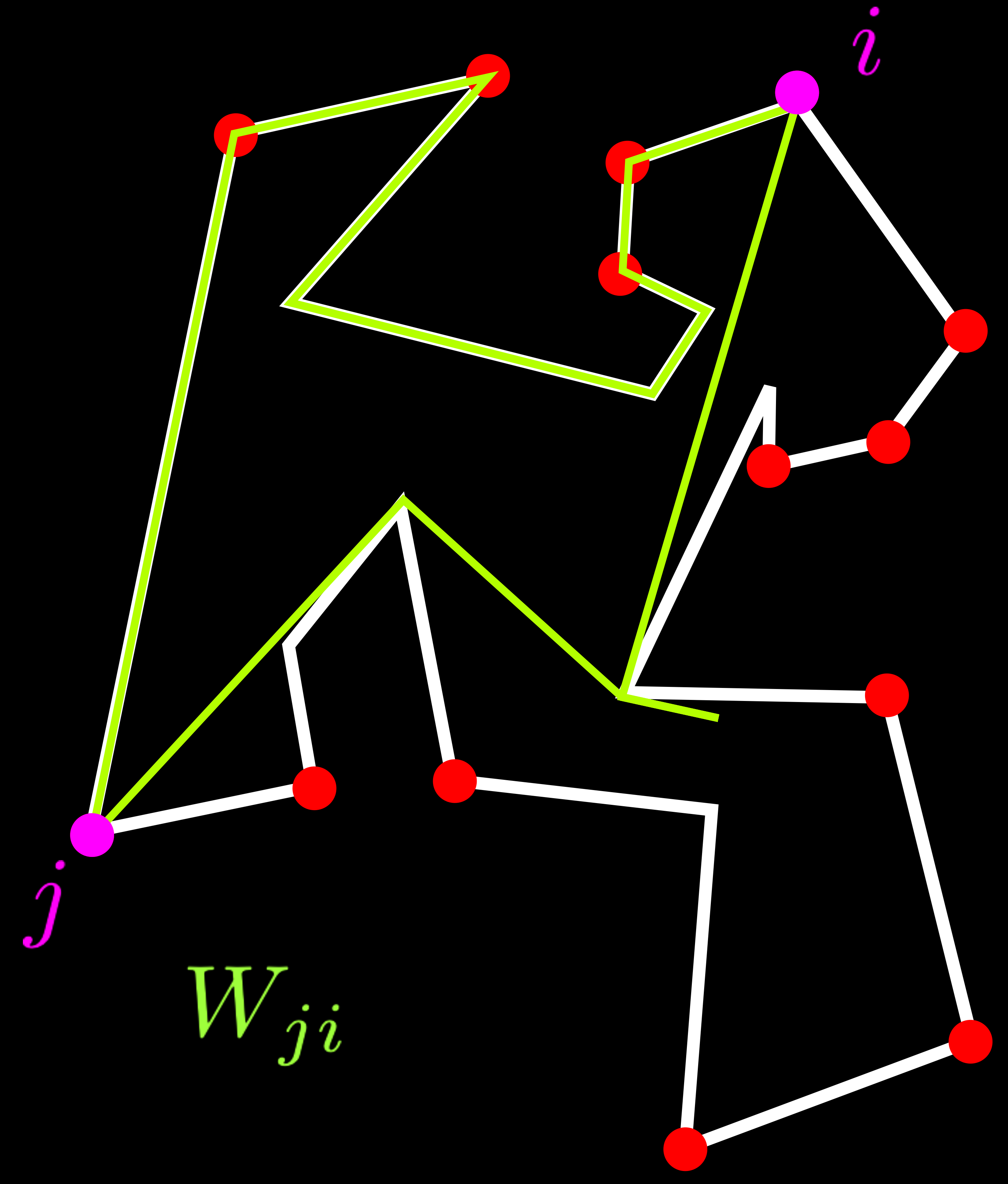
- $C_P$ tour that **visits** all convex vertices



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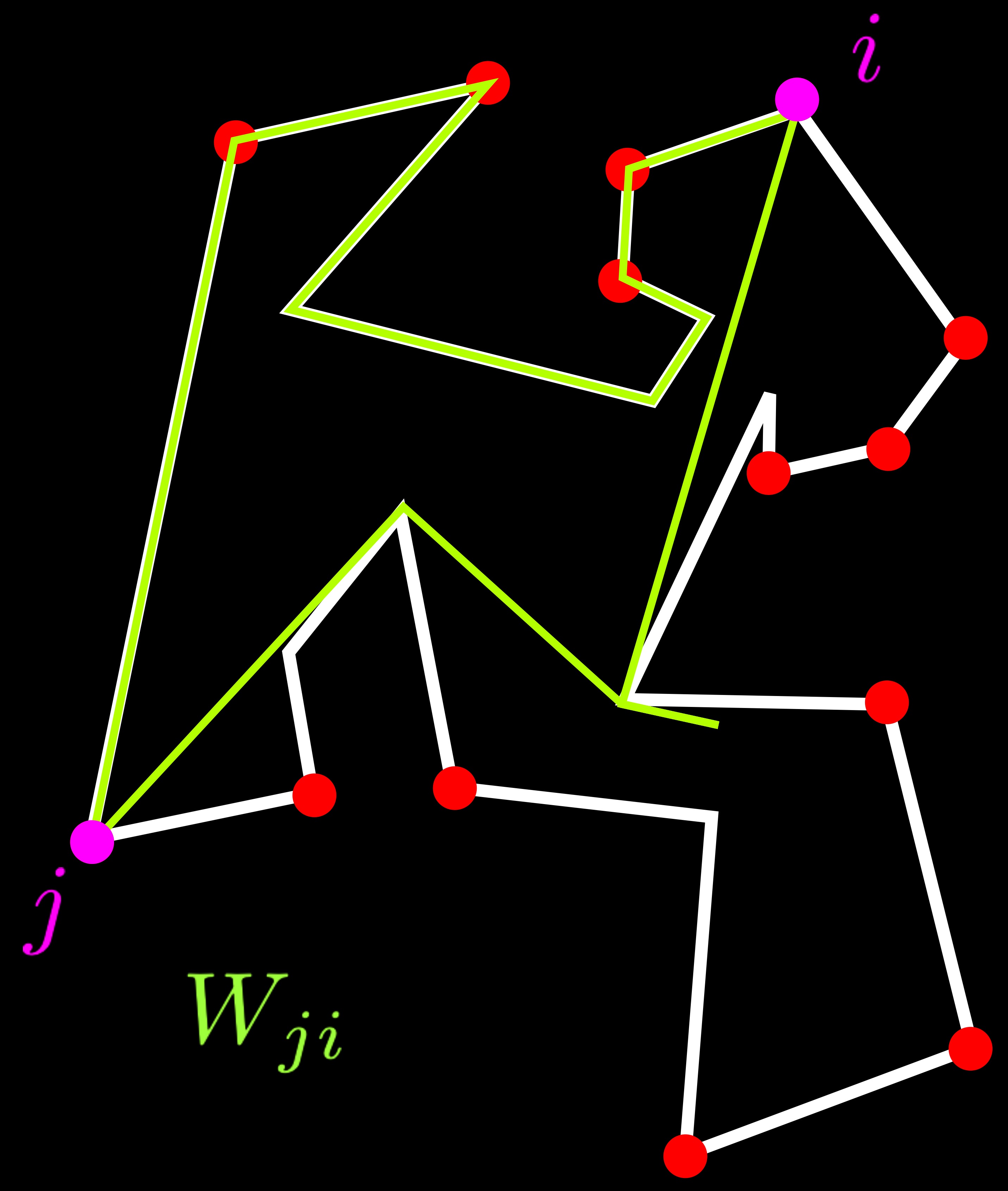

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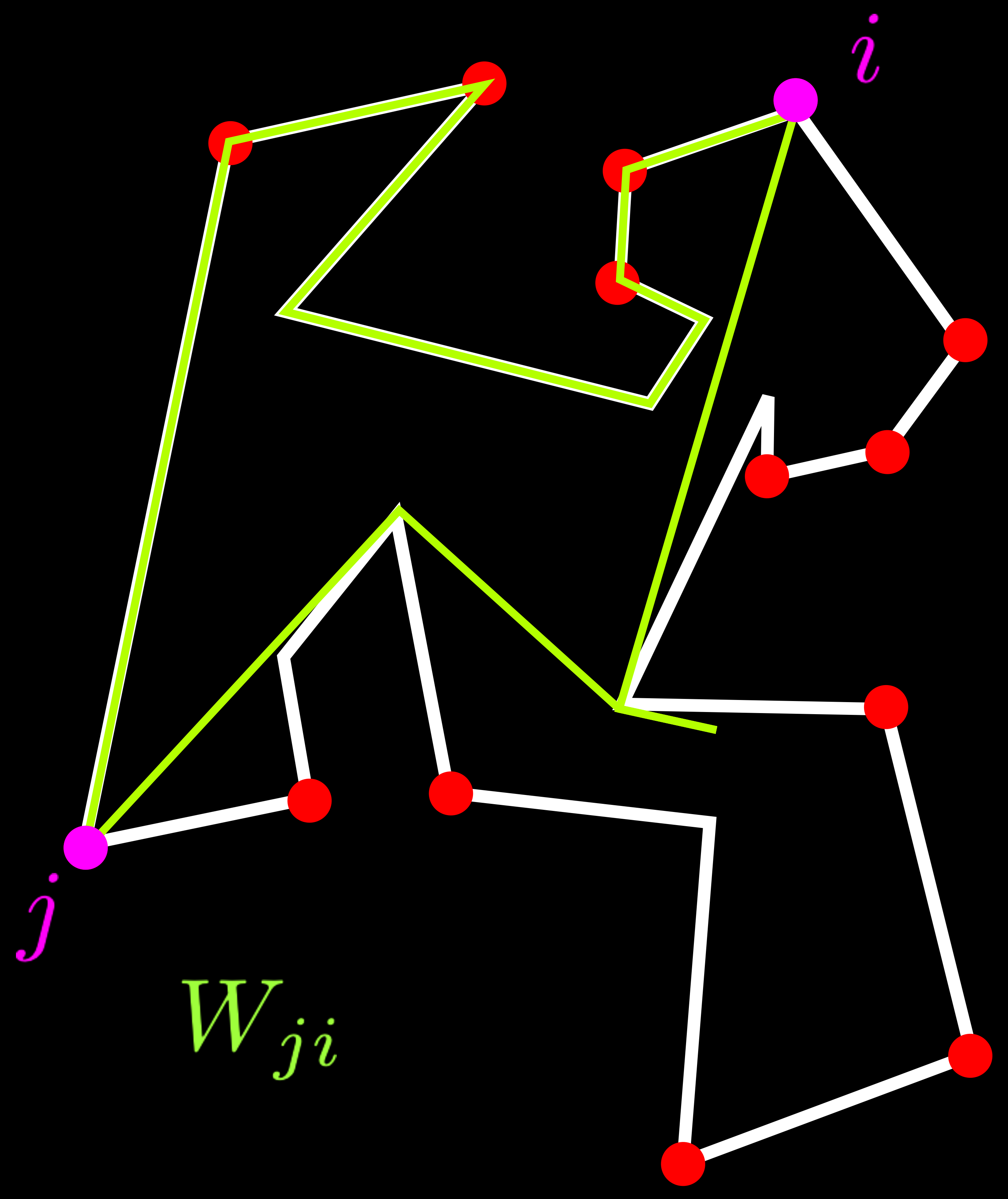
- $C_P$ tour that **visits** all convex vertices
- $D_P$ tour that **sees** all convex vertices
- Our approximation: $(W_1, W_2) = \arg\min_{i \neq j} \{\max\{|W_{ij}|,|W_{ji}|\}, \max\{|C_P|,|D_P|\}\}$




E.g., in case one of the optimal tours visits all convex vertices

# Outlook

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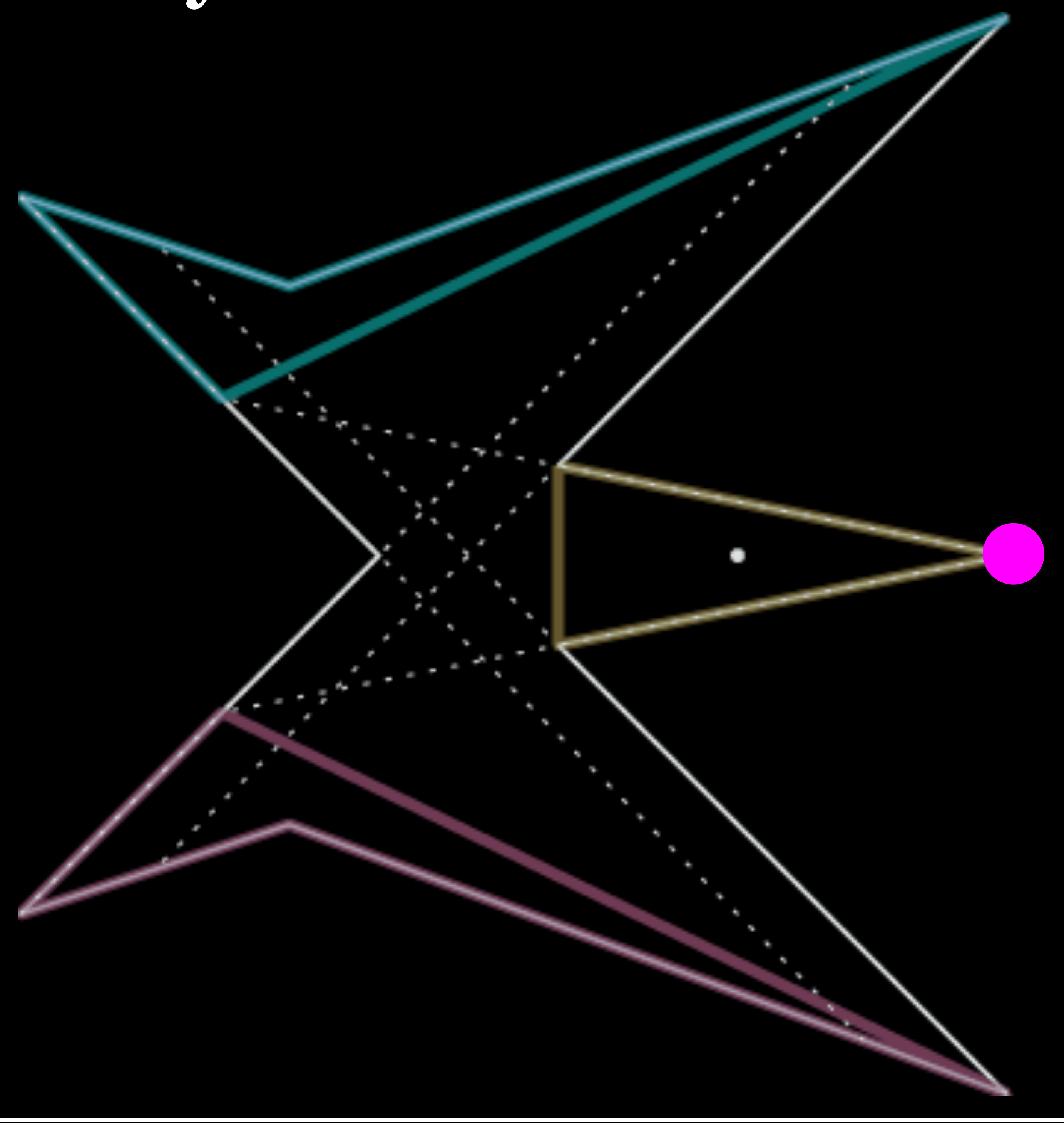
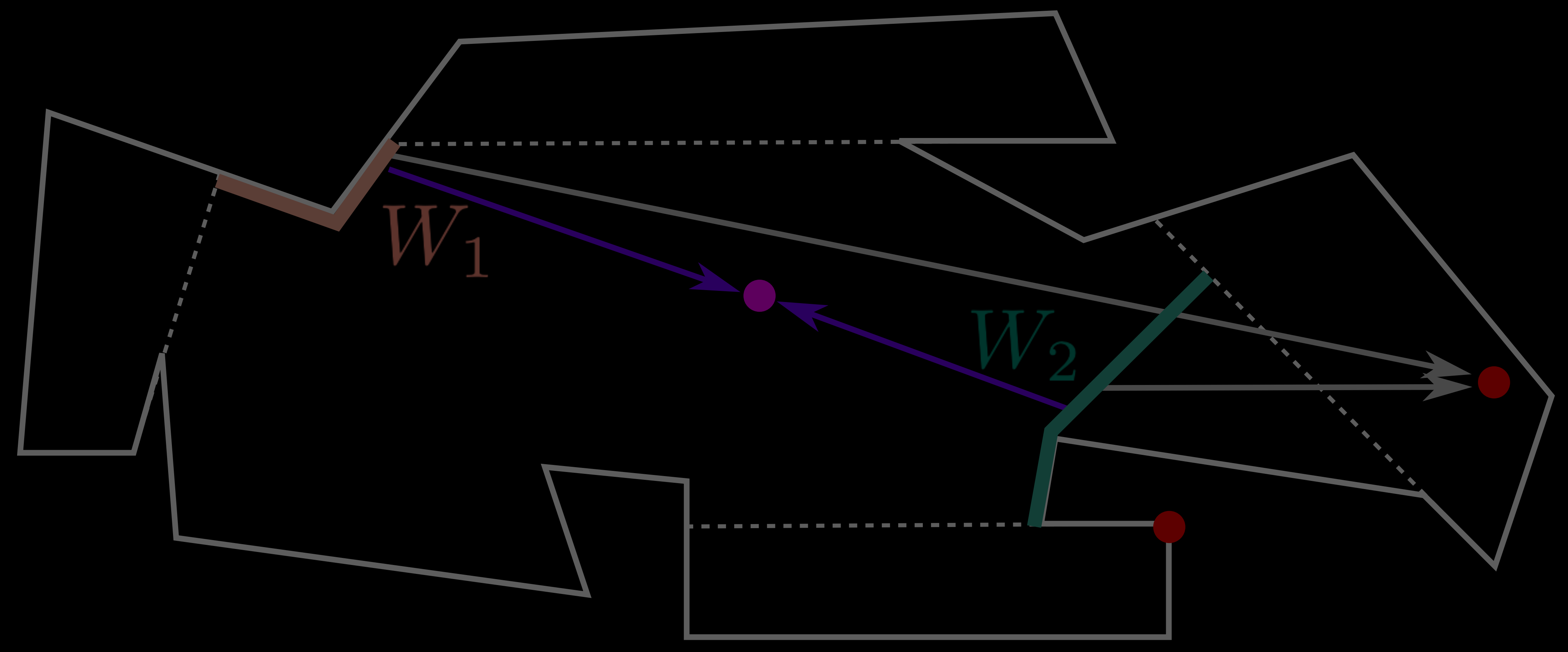
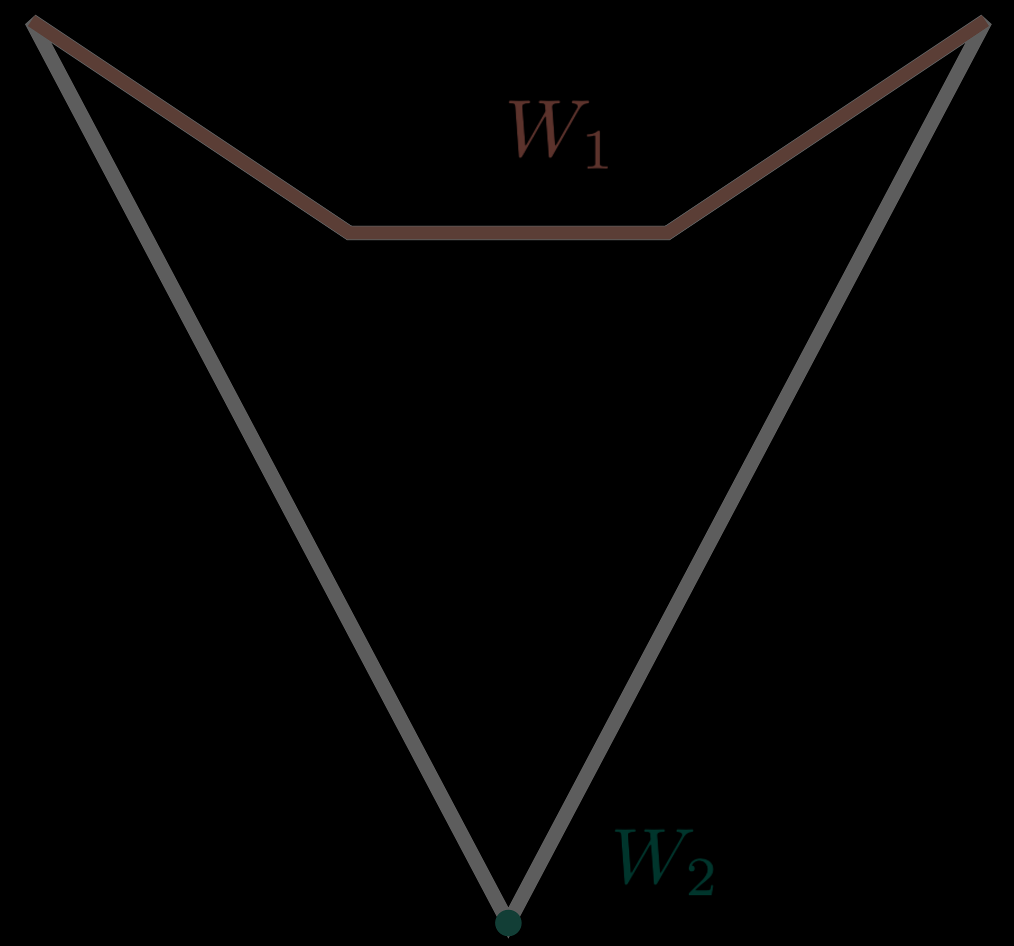
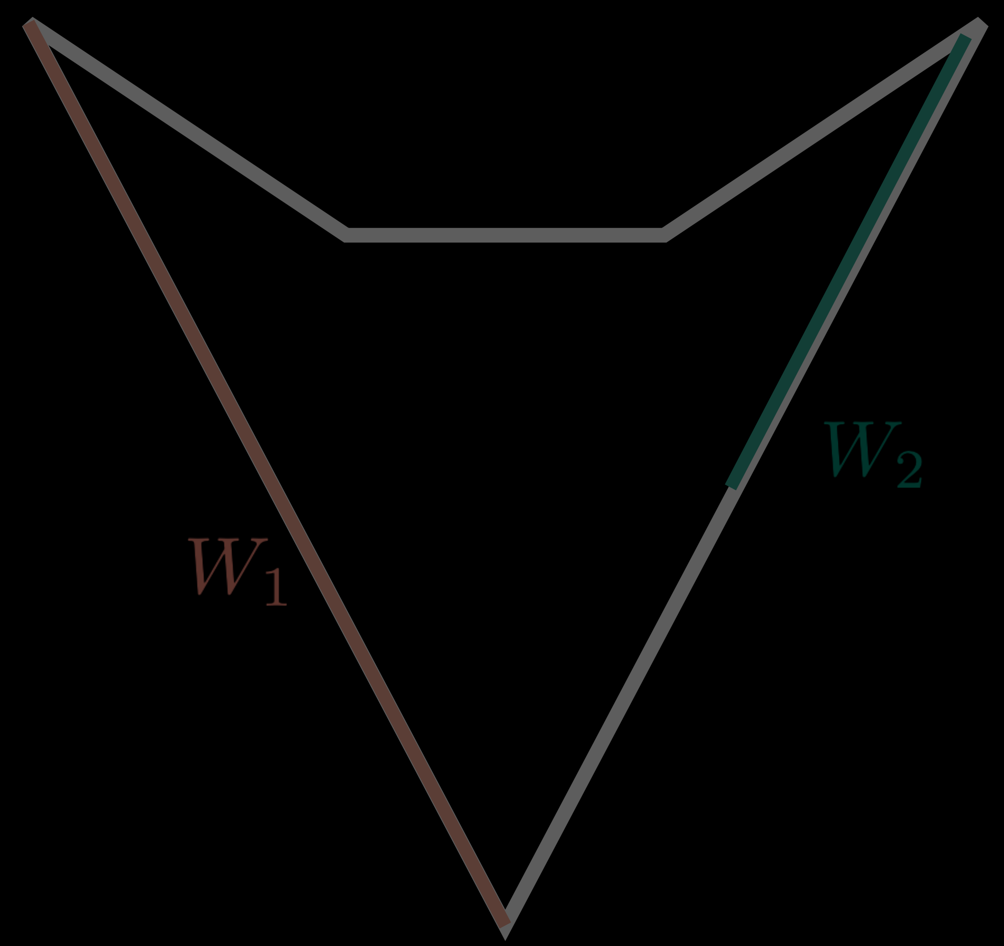
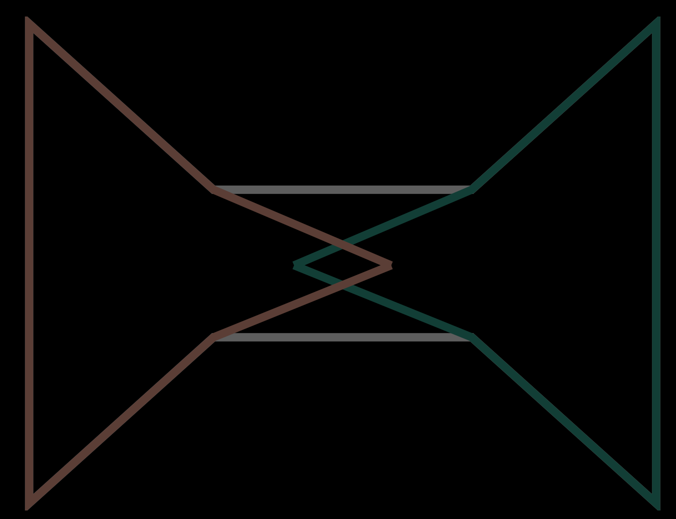
- Is the min-sum version NP-hard?
- Triangle-guarded points if the triangle must also be fully in $P$?

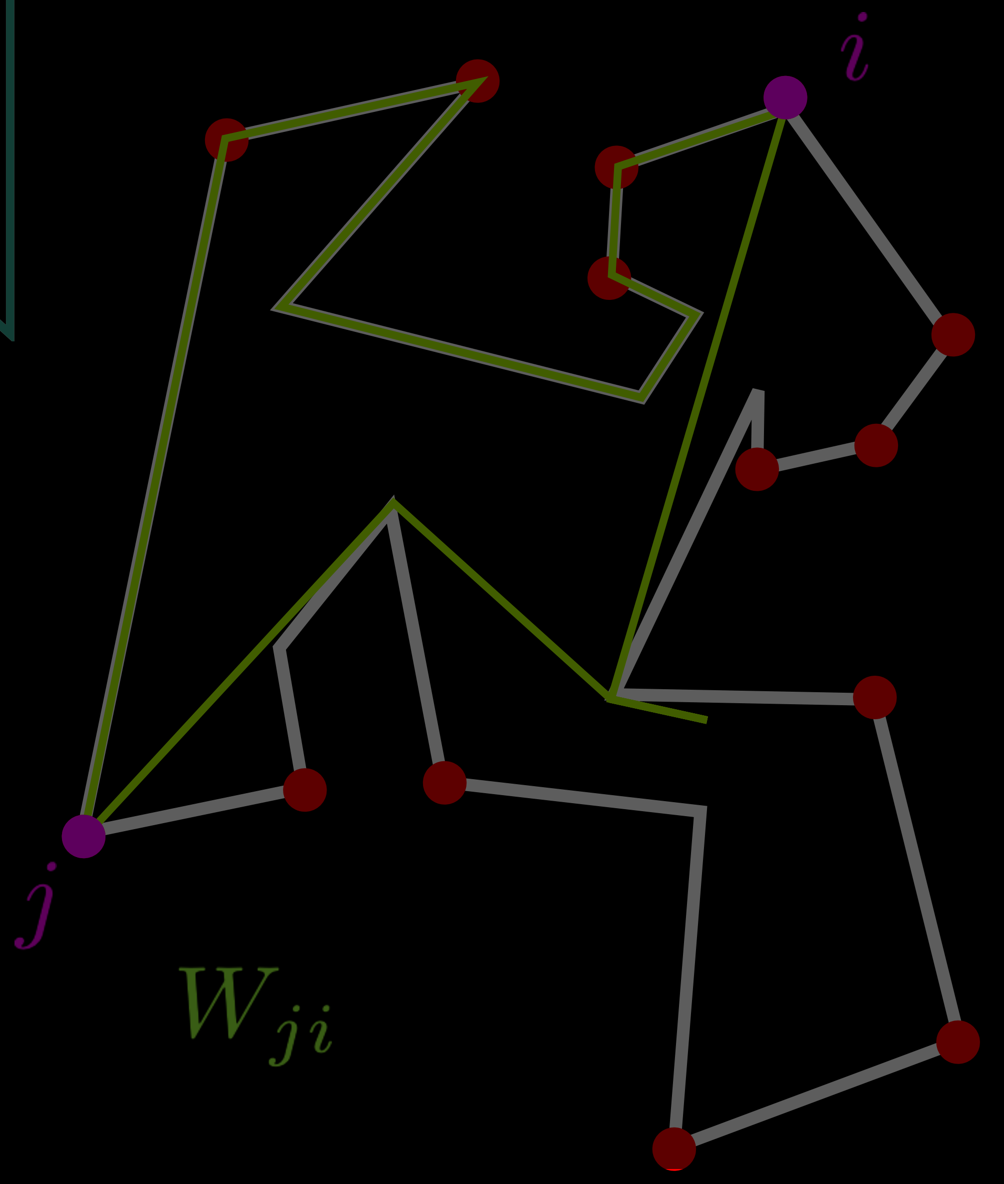




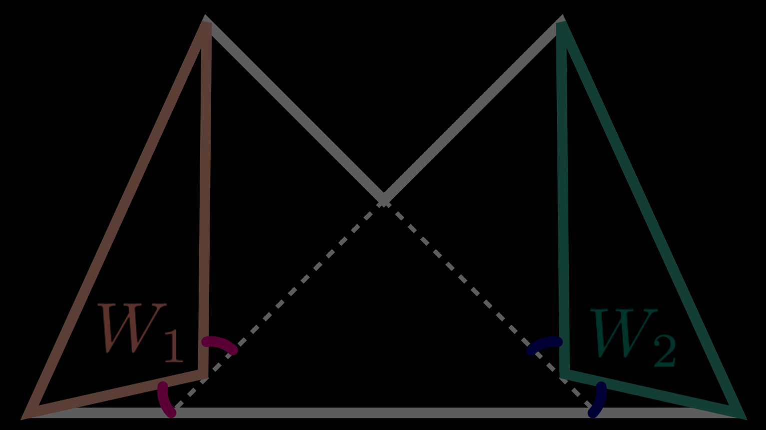



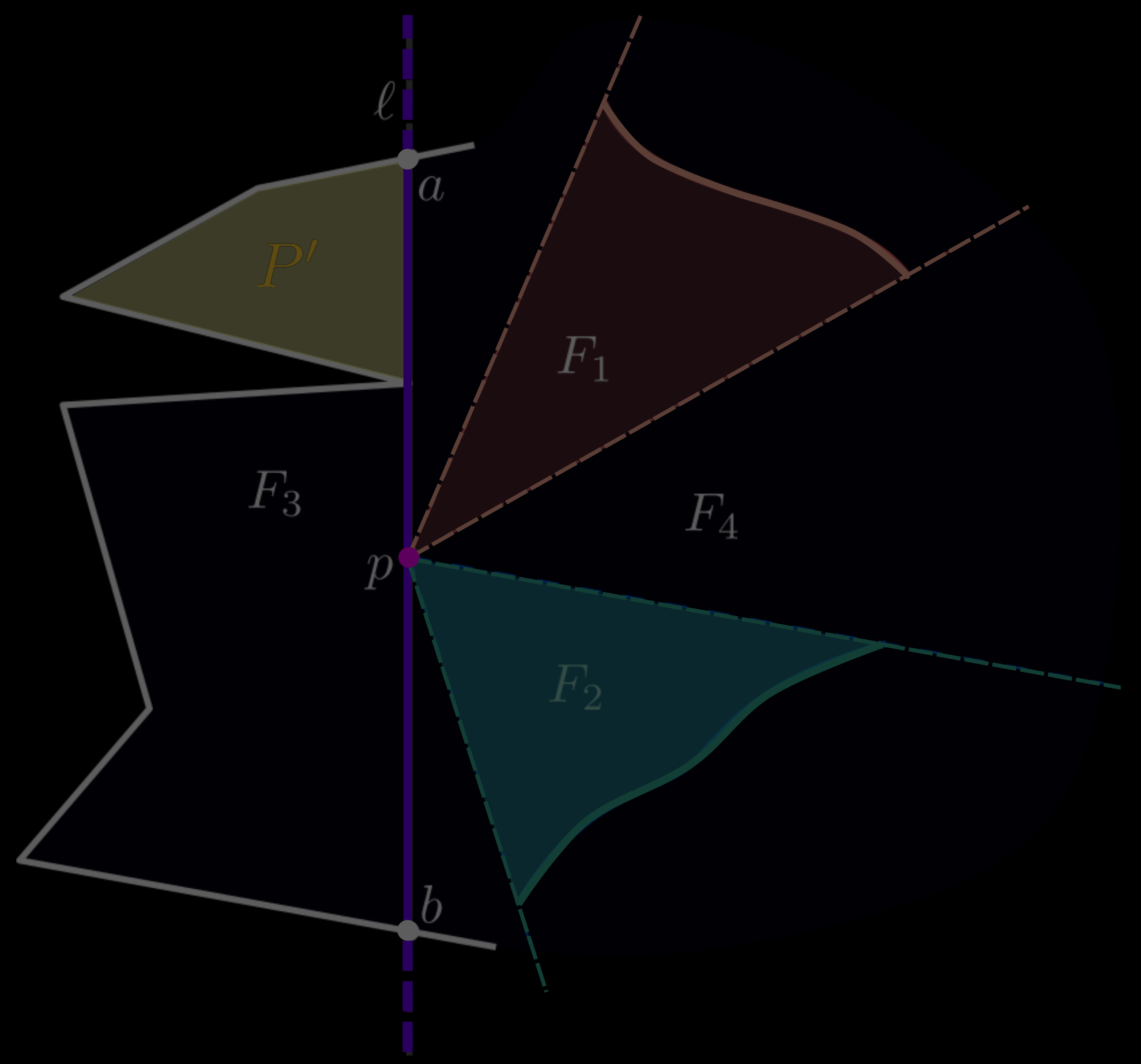






# Thank you.

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