

Shift Scheduling for Air Traffic Controllers in Remote Tower Centers and for Train Dispatchers

Christiane Schmidt, LiU

Based on joint work with Tobias Andersson Granberg, Eulalia Hernández-Romero, Jörn Jakobi, Billy Josefsson, Anastasia Lemetti, Tomas Lidén, Lothar Meyer, Anne Papenfuss, Maximilian Peukert, Tatiana Polishchuk, Valentin Polishchuk, Leonid Sedov, Rabii Zahir

MAI Optimization Seminar, March 4, 2024

- ATCO work
 - Remote Tower Centers (RTCs)
 - Assigning Airports to Remote-Tower Modules
 - ATCO Rostering at an RTC
 - ATCO Workload Investigation
 - Weather Impact on ATCO Work
- Shift Scheduling for Train Dispatchers
 - Problem Formulation
 - First Model
 - Improved Model + Handling of Handovers
- Outlook: Current and Future Work

Remote Tower Center (RTC)



Remote Tower Center (RTC)



- Provides air traffic services to small airports



Remote Tower Center (RTC)



- Provides air traffic services to small airports
- Replaces local tower with cameras and sensors



Remote Tower Center (RTC)



- Provides air traffic services to small airports
- Replaces local tower with cameras and sensors
- Increases efficiency: HR and ATS costs are split between several airports



Remote Tower Center (RTC)



- Provides air traffic services to small airports
- Replaces local tower with cameras and sensors
- Increases efficiency: HR and ATS costs are split between several airports
- First RTC: Sundsvall RTC



Remote Tower Center (RTC)



- Provides air traffic services to small airports
- Replaces local tower with cameras and sensors
- Increases efficiency: HR and ATS costs are split between several airports
- First RTC: Sundsvall RTC
- New RTC build in Stockholm



Remote Tower Center

- How to **distribute** the **total taskload** from several airports over several air traffic controller (ATCO) working positions?



Remote Tower Center

- How to **distribute** the **total taskload** from several airports over several air traffic controller (ATCO) working positions?
- How to assign a qualified **ATCO** to each position, respecting the constraints on the duration of controllers shifts, breaks and the necessity of maintaining ratings?



Remote Tower Center

- How to **distribute** the **total taskload** from several airports over several air traffic controller (ATCO) working positions?
- How to assign a qualified **ATCO** to each position, respecting the constraints on the duration of controllers shifts, breaks and the necessity of maintaining ratings?
- ➔ Development of general optimization framework designed as a flexible tool for **staff planning/shift scheduling**



Remote Tower Center (RTC)

- Which factors contribute to **ATCO's workload**?



Remote Tower Center (RTC)

- Which factors contribute to **ATCO's workload**?
- **Difference** in workload at **RTC vs. conventional towers**?



Remote Tower Center (RTC)

- Which factors contribute to **ATCO's workload**?
- **Difference** in workload at **RTC vs. conventional towers**?
- How do we factor in **unscheduled events: weather**?



Remote Tower Center (RTC)

- Which factors contribute to **ATCO's workload**?
- **Difference** in workload at **RTC vs. conventional towers**?
- How do we factor in **unscheduled events: weather**?
- ➔ Study of ATCO workload in RTCs



Remote Tower Center (RTC)

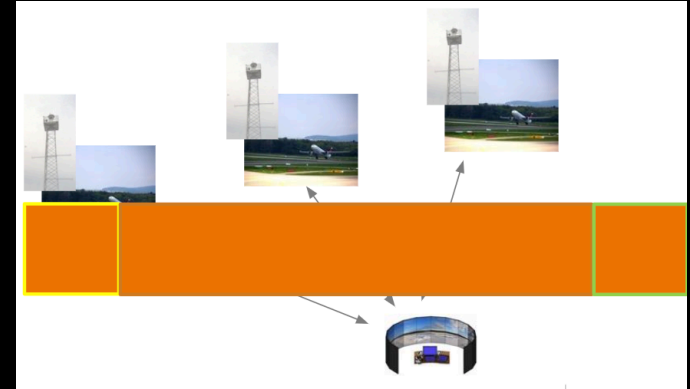
- Which factors contribute to **ATCO's workload**?
- **Difference** in workload at **RTC vs. conventional towers**?
- How do we factor in **unscheduled events: weather**?
- ➔ Study of ATCO workload in RTCs
- ➔ Integration of probabilistic modeling for increased predictability of the extra workload due to unscheduled events (extreme weather conditions) into optimization framework



Assigning Airports to Remote-Tower Modules

Given:

- (1) Example schedules IFR traffic schedules for 1 day (movements = arrival + departure flights) for five Swedish airports
- (2) Specifications of additional special traffic at these airports (military, school, hospital etc.)
- (3) Airport opening hours
- (4) Number of airports assigned to one module bounded (1, 2, 3, ?)
- (5) Total number of moves within a module is bounded



Goal: Propose optimal assignment of the airports to RTC modules

ATCO Rostering at an RTC

ATCO Rostering at an RTC

- Workload from several airports (#movements per airport)

ATCO Rostering at an RTC

- Workload from several airports (#movements per airport)
- Maximum time “in position”

ATCO Rostering at an RTC

- Workload from several airports (#movements per airport)
- Maximum time “in position”
- Scheduled breaks

ATCO Rostering at an RTC

- Workload from several airports (#movements per airport)
- Maximum time “in position”
- Scheduled breaks
- Endorsements and trainings

ATCO Rostering at an RTC

- Workload from several airports (#movements per airport)
- Maximum time “in position”
- Scheduled breaks
- Endorsements and trainings
- 24/7 operation

ATCO Rostering at an RTC

- Workload from several airports (#movements per airport)
- Maximum time “in position”
- Scheduled breaks
- Endorsements and trainings
- 24/7 operation
- Automation needed

ATCO Rostering at an RTC–Objective: min #ATCOs

High traffic



19-Oct-16	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
AP1	0	0	0	0	2	0	0	2	1	0	0	2	0	0	0	0	0	0	0	0	1	0	0	0
AP2	1	1	2	3	4	9	10	7	5	3	2	5	7	4	5	10	8	7	6	8	8	2	0	2
AP3	1	0	2	1	6	5	2	6	4	3	5	4	2	5	6	4	6	8	6	4	3	1	2	2
AP4	0	0	0	0	2	3	3	3	2	1	2	3	2	2	2	4	3	3	0	2	0	0	0	0
AP5	0	0	0	0	3	2	0	4	3	1	2	1	0	2	4	3	2	2	1	2	0	1	0	0

ATCO Rostering at an RTC–Objective: min #ATCOs

High traffic



19-Oct-16	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
AP1	0	0	0	0	2	0	0	2	1	0	0	2	0	0	0	0	0	0	0	0	1	0	0	0
AP2	1	1	2	3	4	9	10	7	5	3	2	5	7	4	5	10	8	7	6	8	8	2	0	2
AP3	1	0	2	1	6	5	2	6	4	3	5	4	2	5	6	4	6	8	6	4	3	1	2	2
AP4	0	0	0	0	2	3	3	3	2	1	2	3	2	2	2	4	3	3	0	2	0	0	0	0
AP5	0	0	0	0	3	2	0	4	3	1	2	1	0	2	4	3	2	2	1	2	0	1	0	0

shifts	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1																								
2																								
3																								
4																								
5																								
6																								
7																								
8																								

Total #. of ATCOs	Av. # of ATCOs per airport	Av. # endorsements per ATCO	Av. time in position	Av. time at work	COP
8	3.4	2.13	7.5	9.38	0.8

Minimum 8
ATCOs needed
to cover 5
airports

ATCO Rostering at an RTC–Objective: min #ATCOs

ATCO	Mon 03.10	Tue 04.10	Wed 05.10	Thur 06.10	Fri 07.10	Sat 08.10	Sun 09.10
1	14:00 18:00	6:00 10:00		23:00 0:00	0:00-8:00 23:00-0:00	0:00-6:00 22:00-0:00	6:00-9:00 23:00-8:00
2		20:00 23:00	0:00-8:00 23:00-0:00	0:00-9:00 20:00-0:00	0:00-6:00 14:00-17:00		
3		8:00 12:00		9:00 17:00	15:00 23:00	16:00 22:00	6:00 16:00
4	19:00 0:00	0:00-2:00 22:00-0:00	0:00-6:00 19:00-0:00	0:00 6:00	19:00 0:00	11:00 16:00	
5		9:00 14:00	7:00 19:00	8:00 20:00	7:00 11:00		
6	0:00-8:00 16:00-21:00	6:00 12:00	13:00 21:00	8:00 17:00			
7	8:00 14:00	6:00 18:00		19:00 23:00	7:00 16:00		
8		15:00 22:00	8:00 12:00	16:00 23:00	9:00 14:00		11:00 23:00
9		20:00 0:00	0:00-7:00	14:00 18:00	10:00 14:00		
10	19:00 0:00	0:00 6:00			19:00 0:00	0:00-7:00 20:00-0:00	0:00-6:00 21:00-0:00
11	8:00 22:00	6:00 18:00	7:00 15:00		11:00 15:00		
12	0:00 9:00		10:00 21:00	6:00 10:00	6:00 18:00		
13	19:00 0:00	0:00 6:00		6:00 18:00			
14	11:00 18:00		20:00 0:00	0:00-8:00 20:00-0:00	17:00 21:00	6:00 15:00	
15	11:00 15:00	10:00 14:00	10:00 14:00			7:00 19:00	9:00 21:00
16			8:00 20:00	6:00 18:00			
17		9:00 20:00	17:00 0:00	0:00-1:00 9:00-13:00			

ATCO	Mon 10.10	Tue 11.10	Wed 12.10	Thur 13.10	Fri 14.10	Sat 15.10	Sun 16.10
1		22:00-0:00	0:00-10:00				
2	9:00 15:00	7:00 14:00		18:00 21:00	10:00 16:00	10:00 19:00	6:00 10:00
3			6:00 12:00				
4	20:00 0:00	0:00-7:00 18:00-0:00	0:00 6:00				
5	14:00 23:00		10:00 20:00		14:00 19:00	19:00 0:00	0:00 7:00
6	8:00 15:00		13:00 19:00	7:00 19:00	8:00 15:00		20:00 0:00
7	19:00 0:00	0:00-3:00 20:00-0:00	0:00-1:00 21:00-0:00	0:00-6:00 16:00-20:00	11:00 17:00		19:00 23:00
8	18:00 0:00	9:00 22:00					
9			8:00 14:00		17:00 22:00	6:00 18:00	6:00 18:00
10	0:00-9:00 18:00-22:00	6:00 12:00					
11		14:00 19:00	10:00 21:00	6:00 16:00		7:00 16:00	
12	19:00 0:00	0:00 6:00		11:00 16:00	17:00 22:00	18:00 0:00	0:00-6:00 21:00-0:00
13			19:00 0:00	0:00-7:00 23:00-0:00	0:00 11:00		8:00 20:00
14	14:00 18:00	6:00 18:00					
15	8:00 19:00	19:00 0:00	0:00 6:00				
16		8:00 20:00			8:00 20:00		
17	9:00 13:00	7:00 11:00		20:00 0:00	0:00-8:00 22:00-0:00	0:00 7:00	12:00 19:00

ATCO Rostering at an RTC–Objective: min #ATCOs

ATCO	Mon 03.10	Tue 04.10	Wed 05.10	Thur 06.10	Fri 07.10	Sat 08.10	Sun 09.10
1	14:00 18:00	6:00 10:00		23:00 0:00	0:00-8:00 23:00-0:00	0:00-6:00 22:00-0:00	6:00-9:00 23:00-8:00
2		20:00 23:00	0:00-8:00 23:00-0:00	0:00-9:00 20:00-0:00	0:00-6:00 14:00-17:00		
3		8:00 12:00		9:00 17:00	15:00 23:00	16:00 22:00	6:00 16:00
4	19:00 0:00	0:00-2:00 22:00-0:00	0:00-6:00 19:00-0:00	0:00 6:00	19:00 0:00	11:00 16:00	
5		9:00 14:00	7:00 19:00	8:00 20:00	7:00 11:00		
6	0:00-8:00 16:00-21:00	6:00 12:00	13:00 21:00	8:00 17:00			
7	8:00 14:00	6:00 18:00		19:00 23:00	7:00 16:00		
8		15:00 22:00	8:00 12:00	16:00 23:00	9:00 14:00		11:00 23:00
9		20:00 0:00	0:00-7:00	14:00 18:00	10:00 14:00		
10	19:00 0:00	0:00 6:00			19:00 0:00	0:00-7:00 20:00-0:00	0:00-6:00 21:00-0:00
11	8:00 22:00	6:00 18:00	7:00 15:00		11:00 15:00		
12	0:00 9:00		10:00 21:00	6:00 10:00	6:00 18:00		
13	19:00 0:00	0:00 6:00		6:00 18:00			
14	11:00 18:00		20:00 0:00	0:00-8:00 20:00-0:00	17:00 21:00	6:00 15:00	
15	11:00 15:00	10:00 14:00	10:00 14:00			7:00 19:00	9:00 21:00
16			8:00 20:00	6:00 18:00			
17		9:00 20:00	17:00 0:00	0:00-1:00 9:00-13:00			
ATCO	Mon 10.10	Tue 11.10	Wed 12.10	Thur 13.10	Fri 14.10	Sat 15.10	Sun 16.10
1		22:00-0:00	0:00-10:00				
2	9:00 15:00	7:00 14:00		18:00 21:00	10:00 16:00	10:00 19:00	6:00 10:00
3			6:00 12:00				
4	20:00 0:00	0:00-7:00 18:00-0:00	0:00 6:00				
5	14:00 23:00		10:00 20:00		14:00 19:00	19:00 0:00	0:00 7:00
6	8:00 15:00		13:00 19:00	7:00 19:00	8:00 15:00		20:00 0:00
7	19:00 0:00	0:00-3:00 20:00-0:00	0:00-1:00 21:00-0:00	0:00-6:00 16:00-20:00	11:00 17:00		19:00 23:00
8	18:00 0:00	9:00 22:00					
9			8:00 14:00		17:00 22:00	6:00 18:00	6:00 18:00
10	0:00-9:00 18:00-22:00	6:00 12:00					
11		14:00 19:00	10:00 21:00	6:00 16:00		7:00 16:00	
12	19:00 0:00	0:00 6:00		11:00 16:00	17:00 22:00	18:00 0:00	0:00-6:00 21:00-0:00
13			19:00 0:00	0:00-7:00 23:00-0:00	0:00 11:00		8:00 20:00
14	14:00 18:00	6:00 18:00					
15	8:00 19:00	19:00 0:00	0:00 6:00				
16		8:00 20:00			8:00 20:00		
17	9:00 13:00	7:00 11:00		20:00 0:00	0:00-8:00 22:00-0:00	0:00 7:00	12:00 19:00

Minimum 17
ATCOs needed
to cover 5
airports during
2 weeks

RTC Efficiency Evaluation

NUMBER OF CONTROLLERS	INDIVIDUAL 5 AIRPORTS	SAME 5 AIRPORTS AT RTC
Lower bound for the highest traffic day (October 19, 2016)	17	8
With the buffer of 33% – 45% for the highest traffic day (October 19, 2016)	26–34	12–15

RTC Efficiency Evaluation

NUMBER OF CONTROLLERS	INDIVIDUAL 5 AIRPORTS	SAME 5 AIRPORTS AT RTC
Lower bound for the highest traffic day (October 19, 2016)	17	8
With the buffer of 33% – 45% for the highest traffic day (October 19, 2016)	26–34	12–15

After optimization, the RTC provides savings of 42-55%

ATCO Workload Investigation

- Within CAPMOD project:
Data analysis from Simulations
 - DLR simulation data
 - Sundsvall validation trials (May-June 2019)

ATCO Workload Investigation

- Within CAPMOD project:

Data analysis from Simulations

- DLR simulation data
- Sundsvall validation trials (May-June 2019)

Observations and data collection in conventional towers (field study) + data analysis

- Field study at Bromma airport (March 2019) video-recording, audio, questionnaires

Statistical learning: Subjective vs. objective assessment (workload rating vs. quantitative measures derived from eye tracking and video analysis)

B. Josefsson, J. Jakobi, A. Papenfuss, T. Polishchuk, C. Schmidt, L. Sedov Identification of Complexity Factors for Remote Towers. In SESAR Innovation Days (SID 2018), December 3-5, Salzburg.

B. Josefsson, L. Meyer, M. Peukert, T. Polishchuk, C. Schmidt: Validation of Controller Workload Predictors at Conventional and Remote Towers, In International Conference for Research in Air Transportation (ICRAT 2020), BEST PAPER AWARD

L. Meyer, M. Peukert, T. Polishchuk, C. Schmidt: Investigating Ocular and Head-Yaw Measures as Indicators for Workload and Fatigue under Varying Taskload Conditions, In 10th International Conference on Research in Air Transportation (ICRAT '22)

ATCO Workload Investigation

- Within CAPMOD project:

Data analysis from Simulations

- DLR simulation data
- Sundsvall validation trials (May-June 2019)

Observations and data collection in conventional towers (field study) + data analysis

- Field study at Bromma airport (March 2019) video-recording, audio, questionnaires

Statistical learning: Subjective vs. objective assessment (workload rating vs. quantitative measures derived from eye tracking and video analysis)

- Now running: OWL project (On WorkLoad Measures)

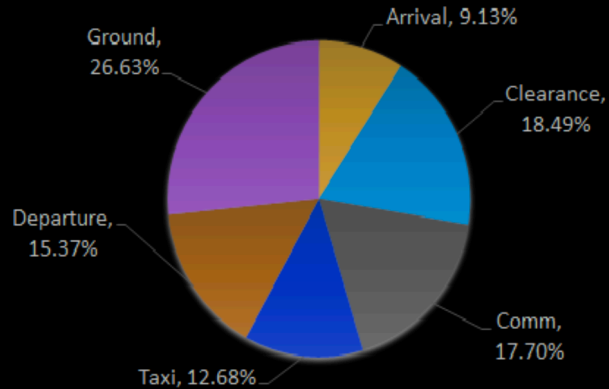
B. Josefsson, J. Jakobi, A. Papenfuss, T. Polishchuk, C. Schmidt, L. Sedov Identification of Complexity Factors for Remote Towers. In SESAR Innovation Days (SID 2018), December 3-5, Salzburg.

B. Josefsson, L. Meyer, M. Peukert, T. Polishchuk, C. Schmidt: Validation of Controller Workload Predictors at Conventional and Remote Towers, In International Conference for Research in Air Transportation (ICRAT 2020), BEST PAPER AWARD

L. Meyer, M. Peukert, T. Polishchuk, C. Schmidt: Investigating Ocular and Head-Yaw Measures as Indicators for Workload and Fatigue under Varying Taskload Conditions, In 10th International Conference on Research in Air Transportation (ICRAT '22)

Bromma Field Study

Weather Impact

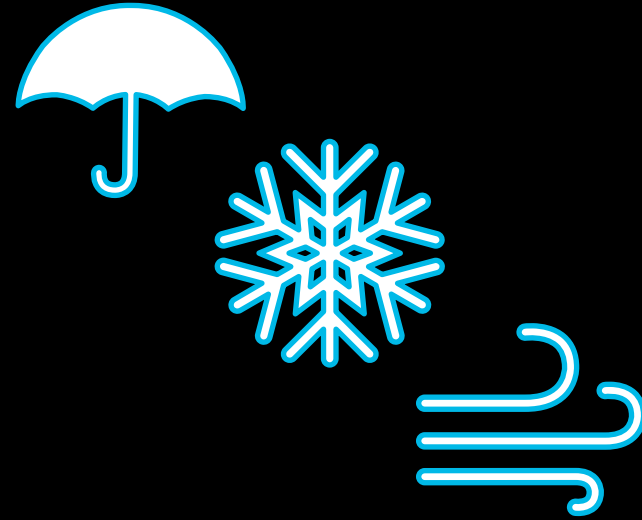


Ground communication takes the largest share in total communication duration

Snow sweeping coordination is a major part in ground communication

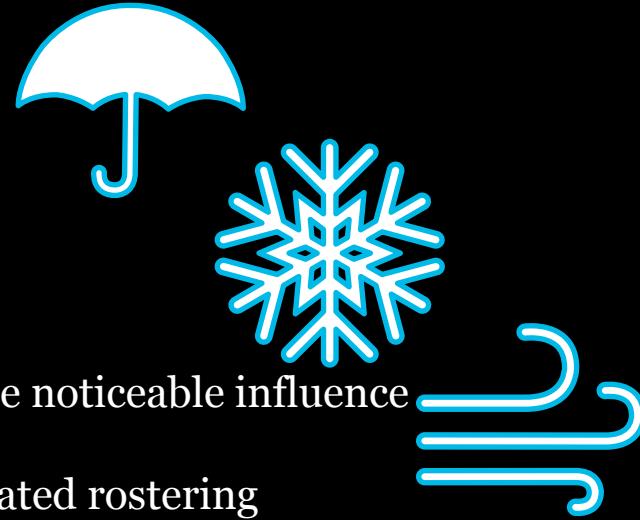
Weather Impact on ATCO Work

- Increased communication with ground services and pilots
- More frequent out-of-window observations
- Changes in Arrival and Departure routes and procedures



Weather Impact on ATCO Work

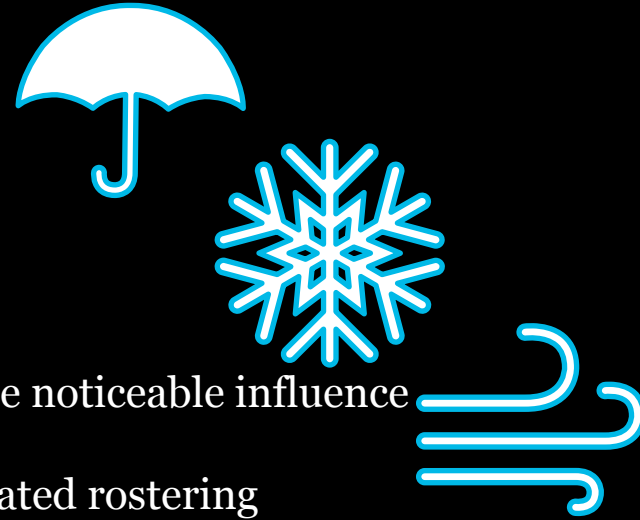
- Increased communication with ground services and pilots
- More frequent out-of-window observations
- Changes in Arrival and Departure routes and procedures
- Conventional towers: staff adjustments are quite rare despite noticeable influence of weather
- Remote towers: weather impact to be integrated into automated rostering
- Multiple operation: ensure no controller is confronted unmanageable workload



Weather Impact on ATCO Work

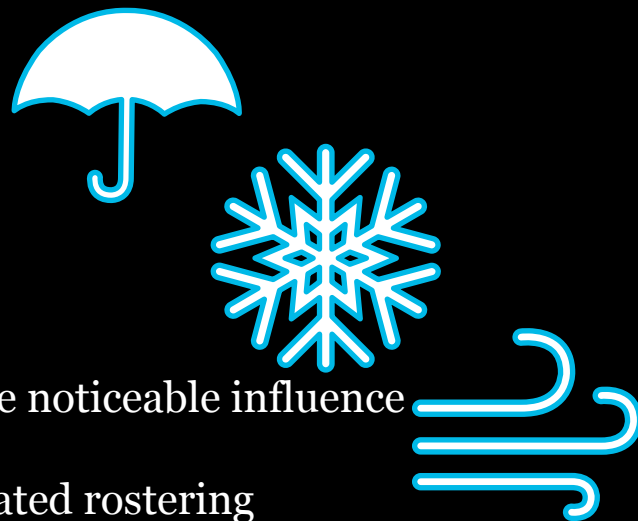
- Increased communication with ground services and pilots
- More frequent out-of-window observations
- Changes in Arrival and Departure routes and procedures
- Conventional towers: staff adjustments are quite rare despite noticeable influence of weather
- Remote towers: weather impact to be integrated into automated rostering
- Multiple operation: ensure no controller is confronted unmanageable workload

No good measures or classifications for weather impact on ATCOs exist



Weather Impact on ATCO Work

- Increased communication with ground services and pilots
- More frequent out-of-window observations
- Changes in Arrival and Departure routes and procedures
- Conventional towers: staff adjustments are quite rare despite noticeable influence of weather
- Remote towers: weather impact to be integrated into automated rostering
- Multiple operation: ensure no controller is confronted unmanageable workload



No good measures or classifications for weather impact on ATCOs exist

Research questions:

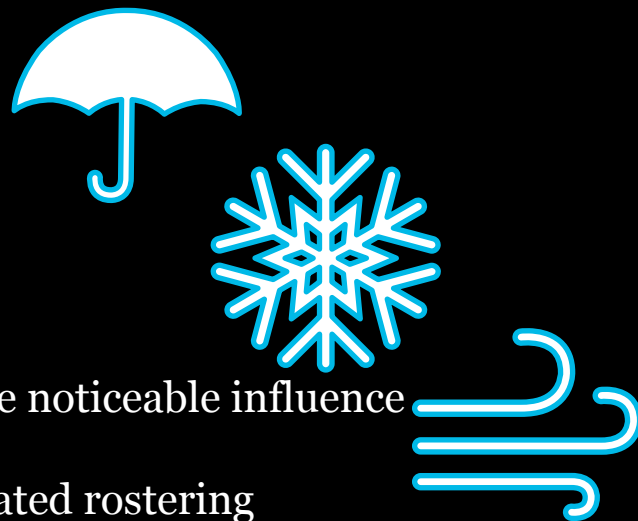
How do different **weather phenomena** impact ATCO workload at different airports?

How to **quantify the weather-induced capacity reductions**?

How can we **integrate this impact in RTC staff scheduling**?

Weather Impact on ATCO Work

- Increased communication with ground services and pilots
- More frequent out-of-window observations
- Changes in Arrival and Departure routes and procedures
- Conventional towers: staff adjustments are quite rare despite noticeable influence of weather
- Remote towers: weather impact to be integrated into automated rostering
- Multiple operation: ensure no controller is confronted unmanageable workload



No good measures or classifications for weather impact on ATCOs exist

Research questions:

How do different **weather phenomena** impact ATCO workload at different airports?

How to **quantify the weather-induced capacity reductions**?

How can we **integrate this impact in RTC staff scheduling**?

Contributes to: **safety assessment of multiple operation** (required by unions and regulation bodies)

Weather Impact on ATCO Work

To integrate weather impact into RTC staff scheduling we propose these steps:

1. Identify impactful weather phenomena for each considered airport
2. Define threshold values for these impactful weather phenomena
3. Obtain weather data in form of EPS
4. Obtain flight movements data for all airports for the chosen dates.
5. Calculate a distribution of the necessary number of ATCOs for RTC staffing

ATCO interviews, identify additional tasks because of different weather phenomena

Weather Impact on ATCO Work

Prose formulation	Numerical value
no	0
rarely, not too much	0.25
sometimes, maybe, can happen, several times	0.5
often, increased, more likely, higher	0.75
yes	1
much more; yes, significantly	1.25

To integrate weather impact into RTC staff scheduling we propose these steps:

1. Identify impactful weather phenomena for each considered airport
2. Define threshold values for these impactful weather phenomena
3. Obtain weather data in form of EPS
4. Obtain flight movements data for all airports for the chosen dates.
5. Calculate a distribution of the necessary number of ATCOs for RTC staffing

Weather Impact on ATCO Work

To integrate weather impact into RTC staff scheduling we propose these steps:

1. Identify impactful weather phenomena for each considered airport
2. Define threshold values for these impactful weather phenomena
3. Obtain weather data in form of EPS
4. Obtain flight movements data for all airports for the chosen dates.
5. Calculate a distribution of the necessary number of ATCOs for RTC staffing

Snow

Precipitation

Low Visibility

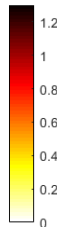
Strong Winds

Convective Activity

We summed up the numerical values reflecting controller's answers and divided by the number of additional tasks

Airport	Snow		
	light	moderate	severe
AP1	0.078	0.57	0.73
AP2	0.046	0.51	0.75
AP3	0.062	0.28	0.6
AP4	0.046	0.046	0.45
AP5	0.14	0.25	0.28

Airport	Convective Activity		
	light	moderate	severe
AP1	0	0	0.75
AP2	0	0	0.75
AP3	0.08	0.11	0.6
AP4	0.17	0.22	0.31
AP5	0.71	0.76	0.84



Weather Impact on ATCO Work

To integrate weather impact into RTC staff scheduling we propose these steps:

1. Identify impactful weather phenomena for each considered airport
2. Define threshold values for these impactful weather phenomena
3. Obtain weather data in form of EPS
4. Obtain flight movements data for all airports for the chosen dates.
5. Calculate a distribution of the necessary number of ATCOs for RTC staffing

Input: Flight movements at several RTC airports per hour + **requirements for single operation**

Output: Optimal assignment of controllers to RTC airports per hour

Constraints: Operational + controller shift constraints (in multiple mode - max 2 a/p per ATCO)

+new constraint to force single operation

Formulated as MILP (mixed-integer linear program)

B. Josefsson, A. Lemetti, T. Polishchuk, V. Polishchuk, C. Schmidt. Integrating Weather Impact in RTC Staff Scheduling. SESAR Innovation Days (SID) 2020.

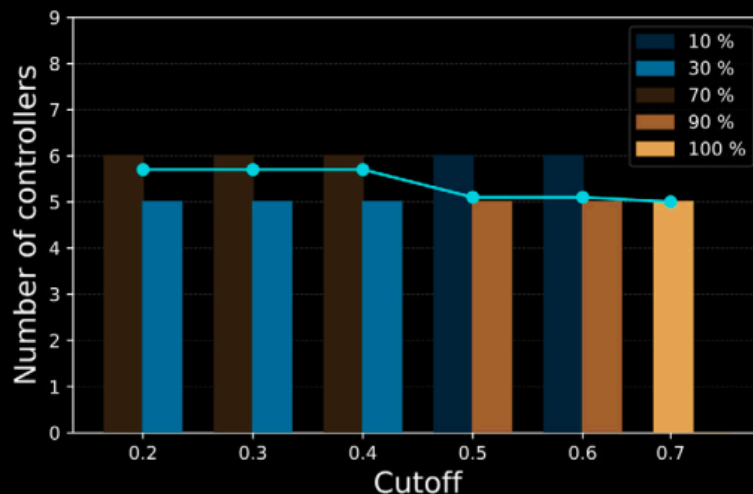
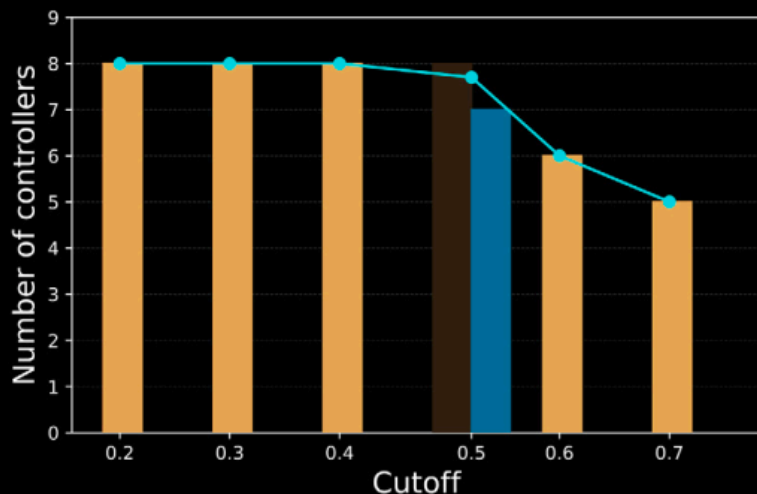
E. Hernández-Romero, B. Josefsson, A. Lemetti, T. Polishchuk, C. Schmidt: Integrating Weather Impact in Air Traffic Controller Shift Scheduling in Remote and Conventional Towers, In EURO Journal on Transportation and Logistics

Weather Impact on ATCO Work

February 16, 2020 (6:00 - 14:00) winter weather

July 29, 2020 (14:00-22:00) summer weather

No weather impact: 5 controllers needed



B. Josefsson, A. Lemetti, T. Polishchuk, V. Polishchuk, C. Schmidt. Integrating Weather Impact in RTC Staff Scheduling. SESAR Innovation Days (SID) 2020.

E. Hernández-Romero, B. Josefsson, A. Lemetti, T. Polishchuk, C. Schmidt: Integrating Weather Impact in Air Traffic Controller Shift Scheduling in Remote and Conventional Towers, In EURO Journal on Transportation and Logistics

Shift Scheduling for Train Dispatchers

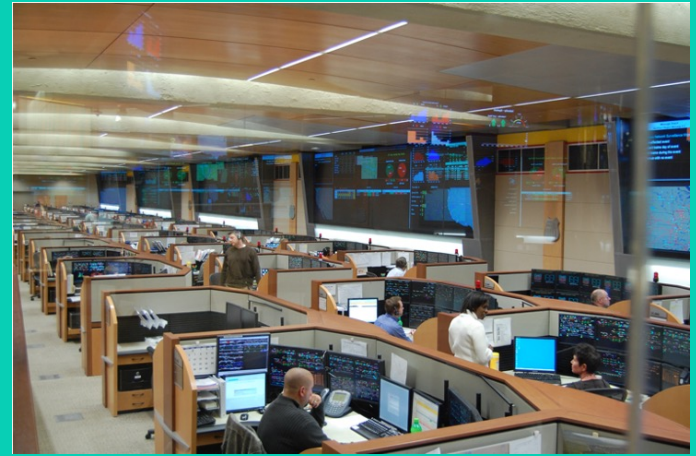


Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue
- High tempo: high workload (WL) level



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue
- High tempo: high workload (WL) level
- What could be improved?



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue
- High tempo: high workload (WL) level
- What could be improved?

Shift scheduling today:



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue
- High tempo: high workload (WL) level
- What could be improved?

Shift scheduling today:

- Manual process



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue
- High tempo: high workload (WL) level
- What could be improved?

Shift scheduling today:

- Manual process
- Very complex and time consuming



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue
- High tempo: high workload (WL) level
- What could be improved?

Shift scheduling today:

- Manual process
- Very complex and time consuming
- Should fulfill many legal and operational constraints



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

Now: Train Dispatchers

- The role of a train dispatcher:
 - Monitoring & controlling train traffic
 - Routing, communication, administration, ...
- Operating 24/7: risk for fatigue
- High tempo: high workload (WL) level
- What could be improved?

Shift scheduling today:

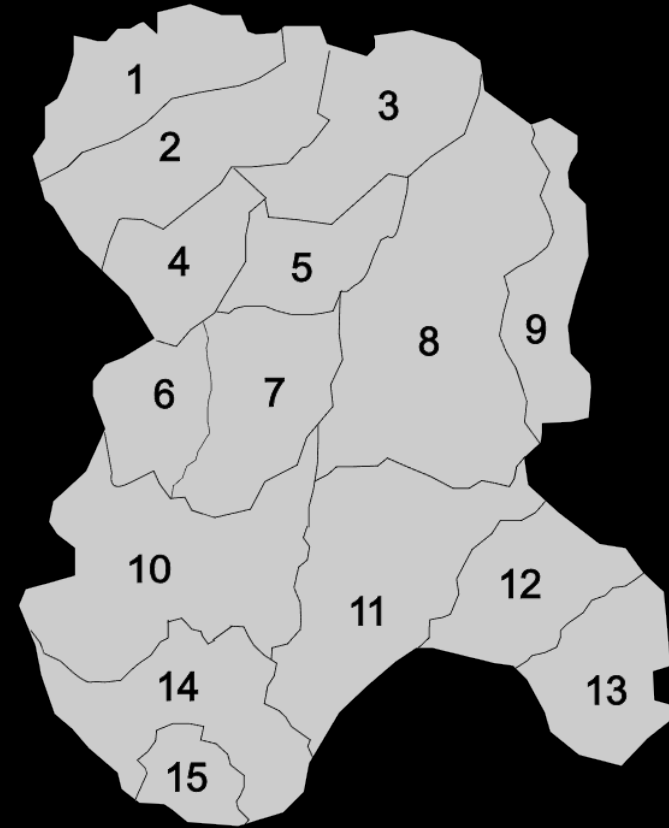
- Manual process
- Very complex and time consuming
- Should fulfill many legal and operational constraints
- May result in over/understaffed shifts: cost vs safety



Image source: <https://www.flickr.com/photos/americaspower/3418746613/>

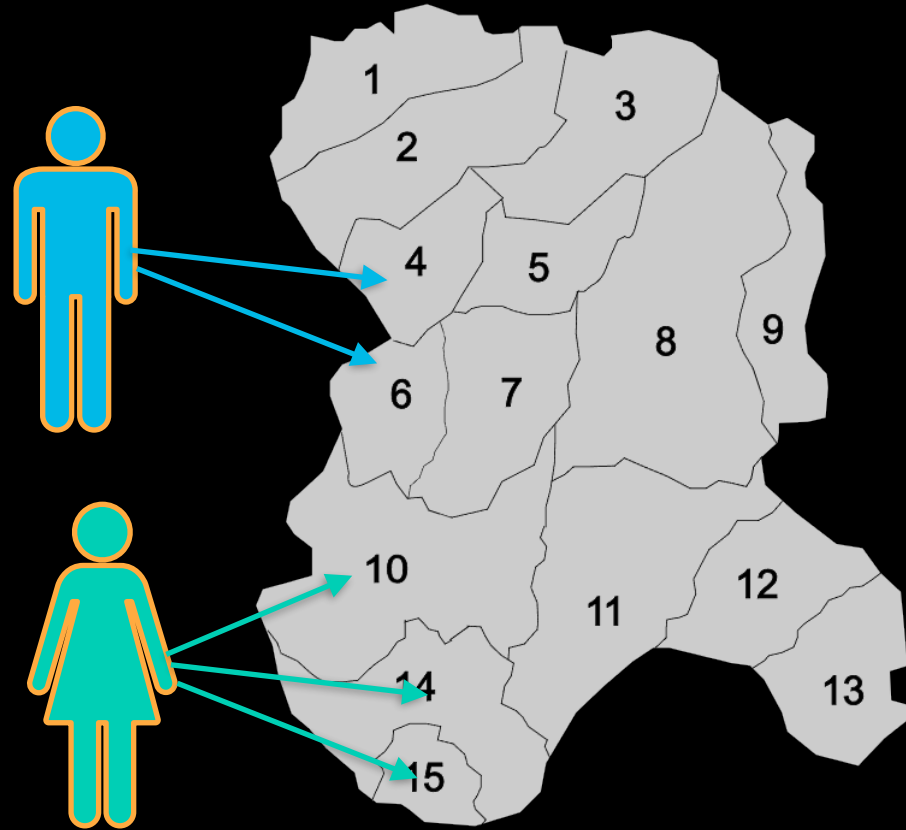
Problem

- A set of geographical areas to cover (for one day)



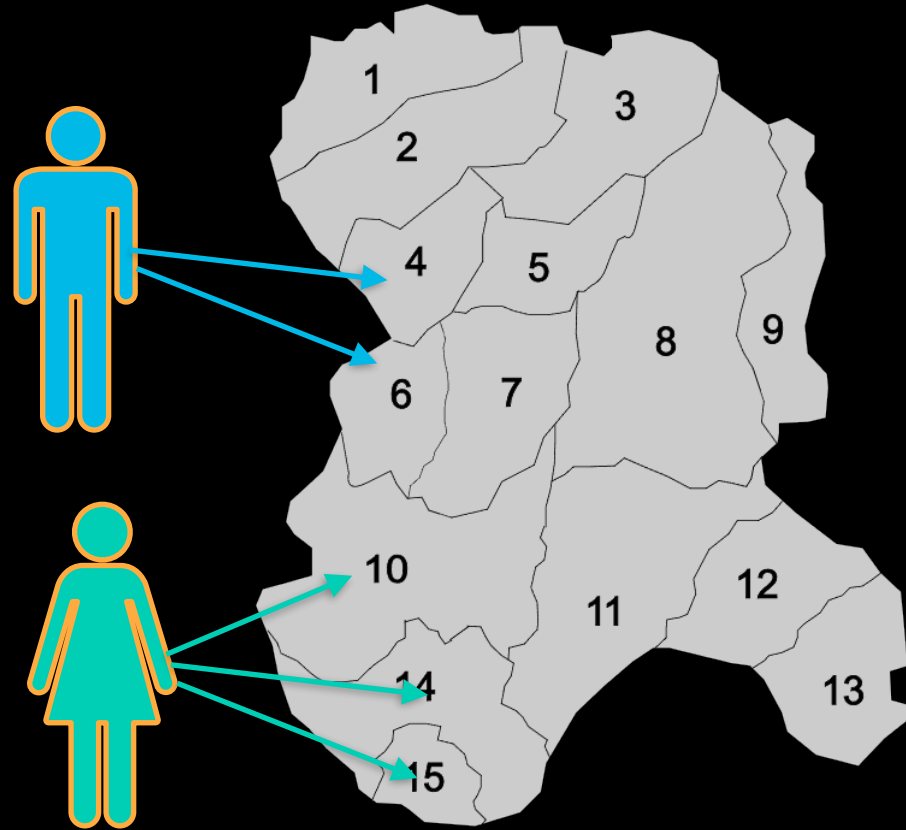
Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas



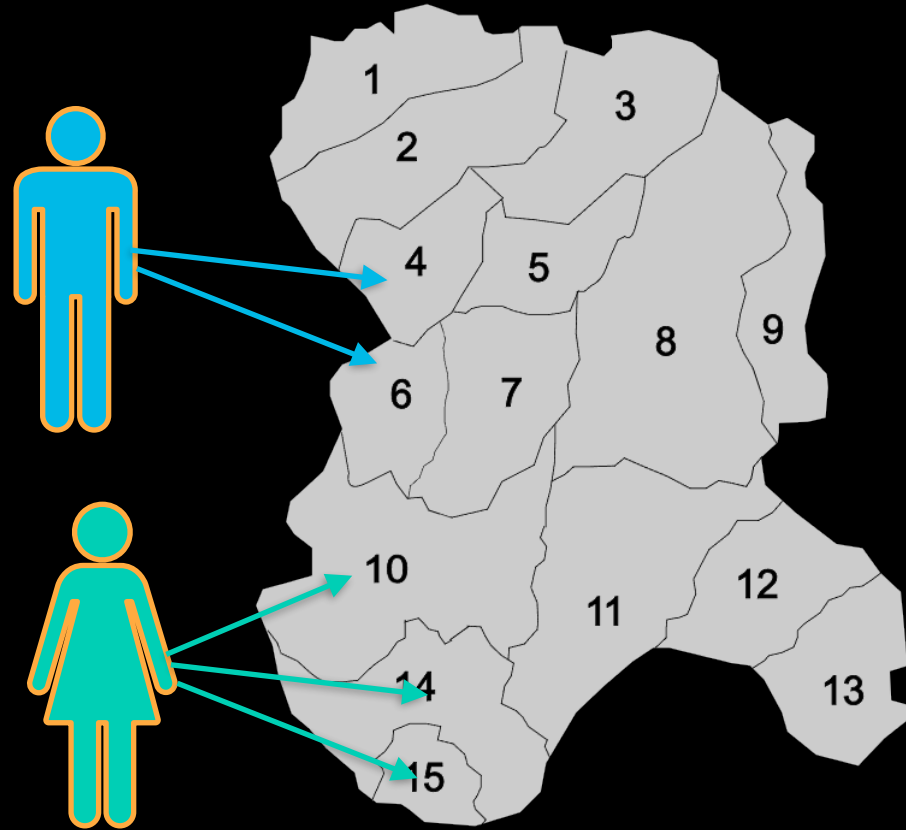
Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas
- Allowed shift length



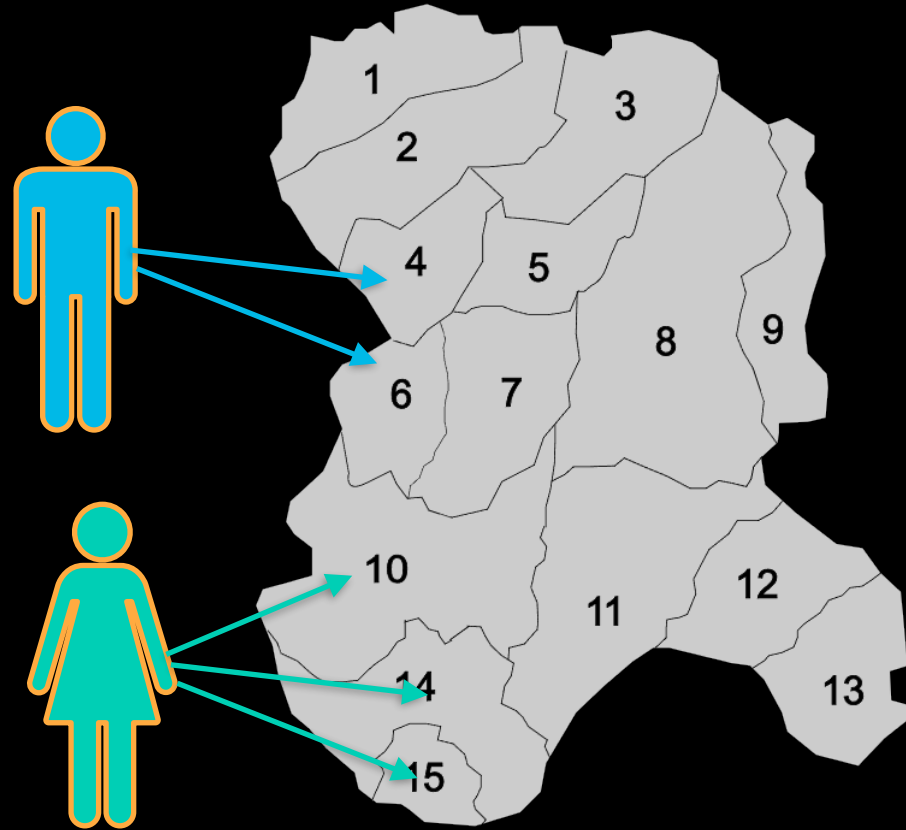
Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas
- Allowed shift length
- Minimum resting time between shifts



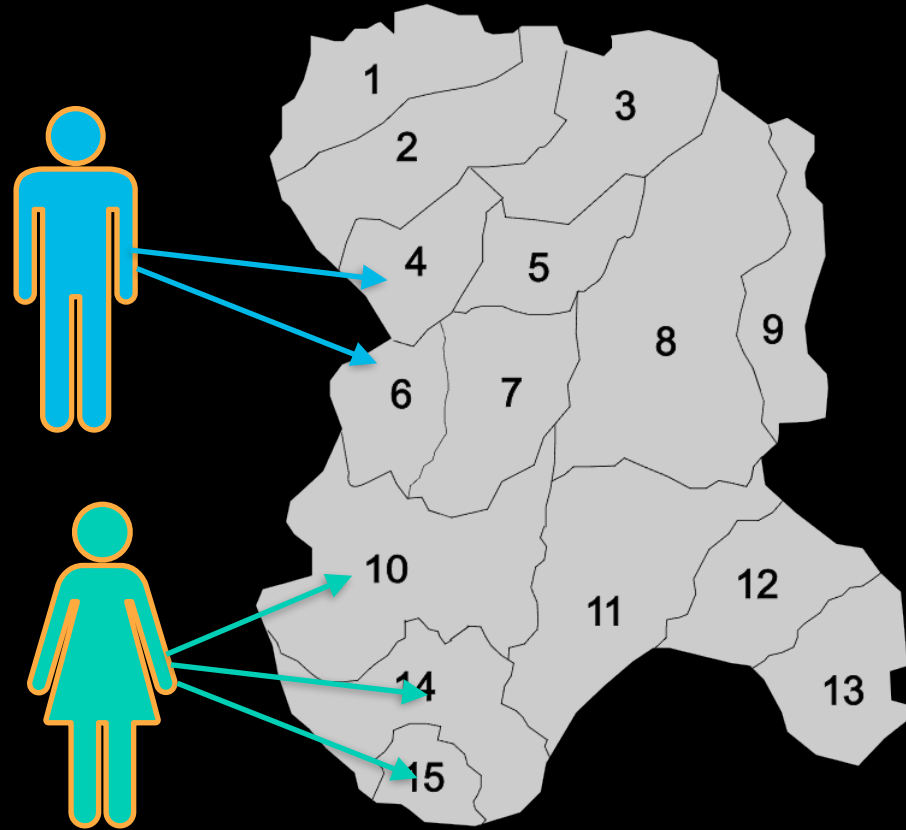
Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas
- Allowed shift length
- Minimum resting time between shifts
- Task load (train movements) per area and time period



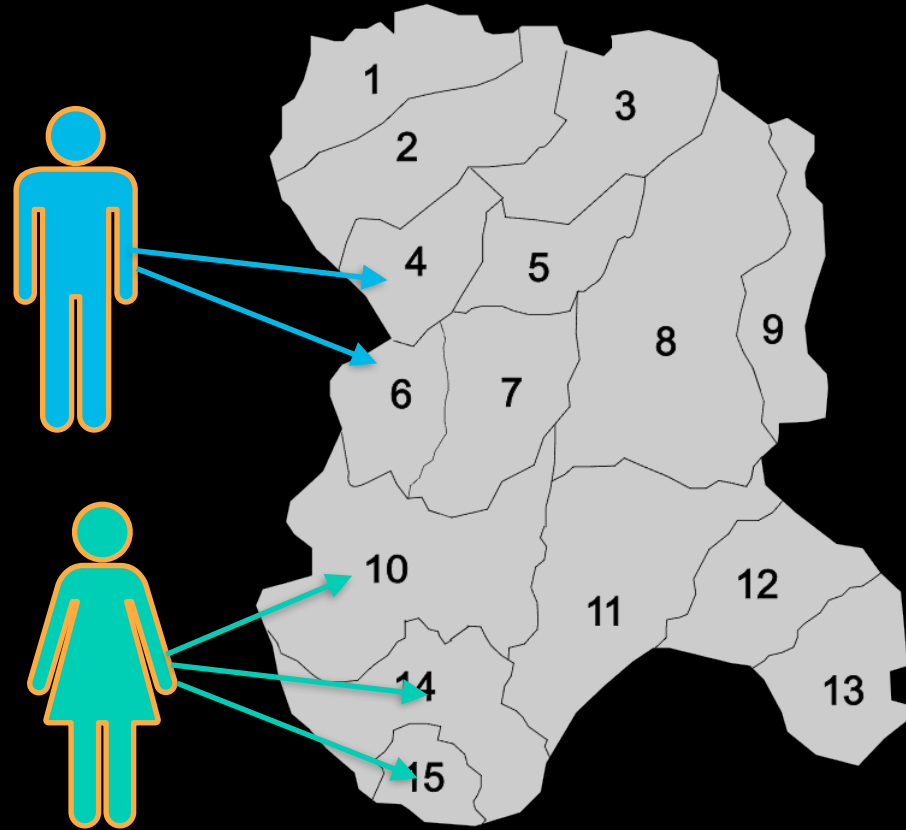
Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas
- Allowed shift length
- Minimum resting time between shifts
- Task load (train movements) per area and time period
- The objective: minimize # of used dispatchers



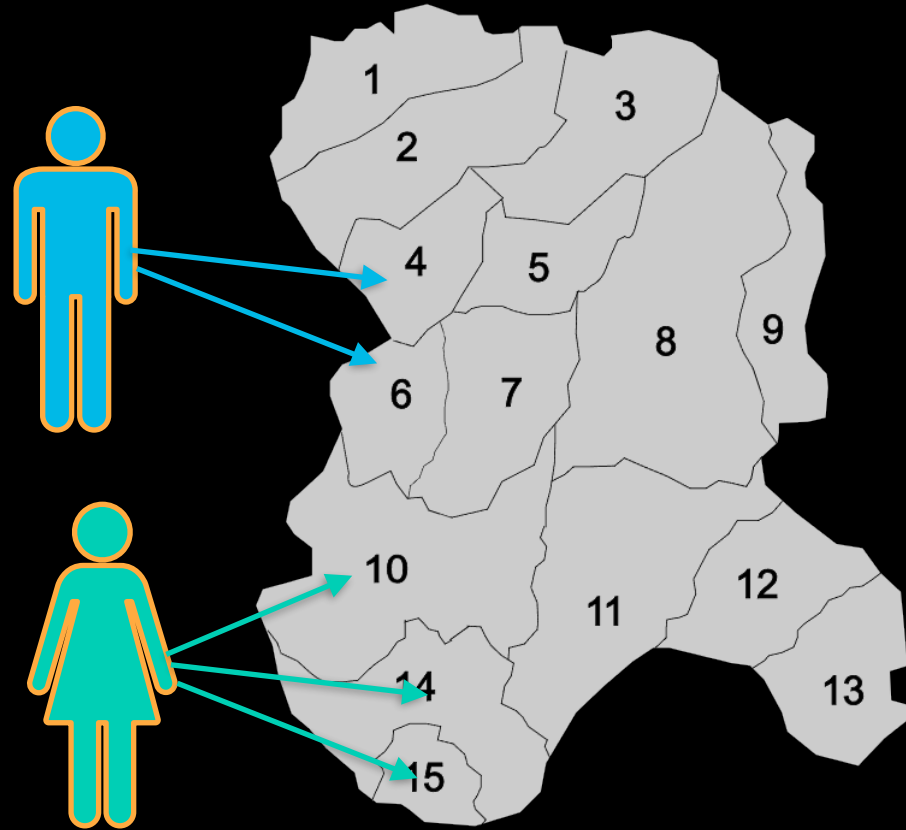
Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas
- Allowed shift length
- Minimum resting time between shifts
- Task load (train movements) per area and time period
- The objective: minimize # of used dispatchers
- Areas can be combined if



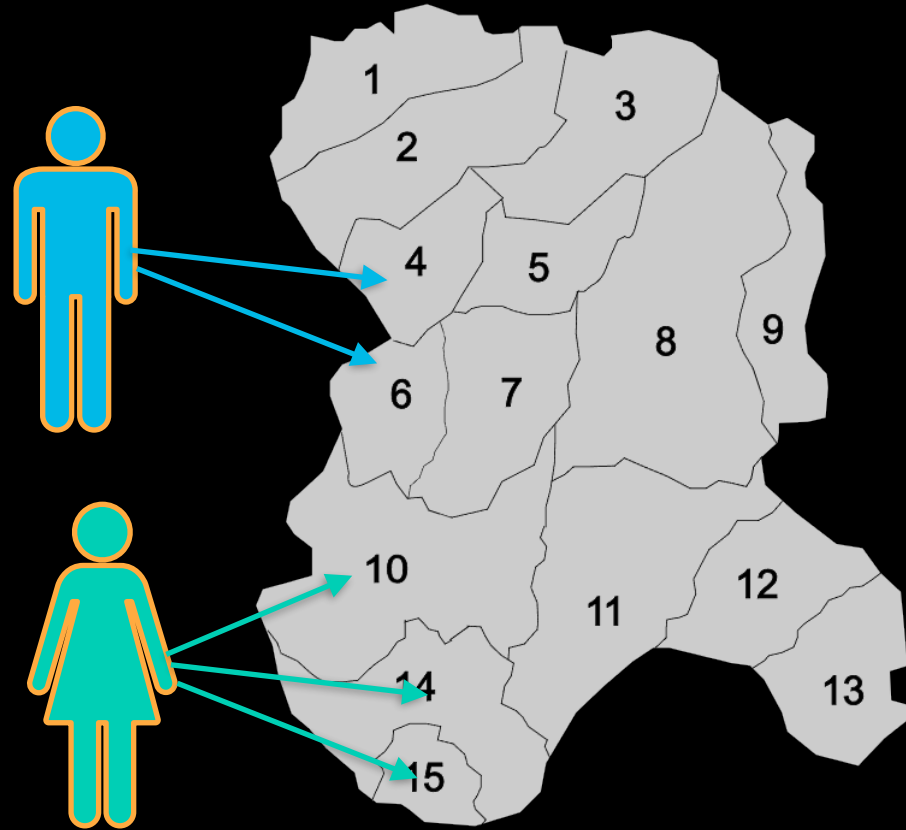
Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas
- Allowed shift length
- Minimum resting time between shifts
- Task load (train movements) per area and time period
- The objective: minimize # of used dispatchers
- Areas can be combined if
 - Taskload allows it



Problem

- A set of geographical areas to cover (for one day)
- A set of dispatchers to be assigned to the areas
- Allowed shift length
- Minimum resting time between shifts
- Task load (train movements) per area and time period
- The objective: minimize # of used dispatchers
- Areas can be combined if
 - Taskload allows it
 - Assigned areas are adjacent



Train Dispatchers: First Model

Variables	Description
$x_{i,j,k} \in \{0, 1\}$	=1 if dispatcher i is assigned area j during period k
$c_{i,\ell,k} \in \{0, 1\}$	=1 if dispatcher i is assigned area combination ℓ during period k
$y_{i,k} \in \{0, 1\}$	=1 if dispatcher i is at work during period k
$v_{i,k} \in \{0, 1\}$	=1 if dispatcher i starts a shift during period k
$q_i \in \{0, 1\}$	=1 if dispatcher i is used during some period

Train Dispatchers: First Model

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k , he must be assigned to some area j
- Minimum rest between shifts

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k , he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod{p}} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod{p}} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod{p}} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod{p}} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC
- If i works during k, they must be assigned to one CC

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Train Dispatchers: First Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC
- If i works during k, they must be assigned to one CC
- Negative connection of x and c

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

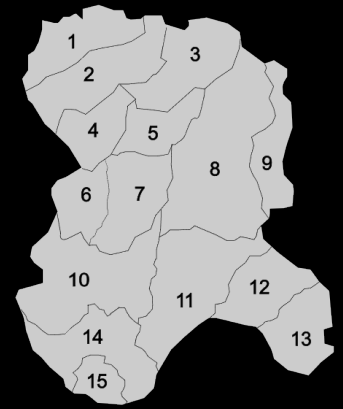
$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

Experiments

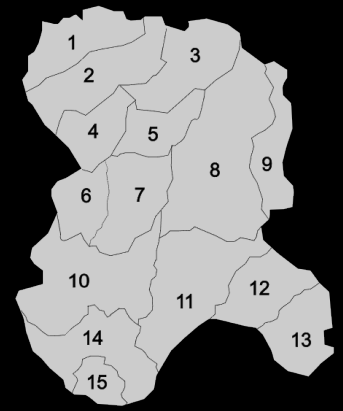
Base scenario:



Experiments

Base scenario:

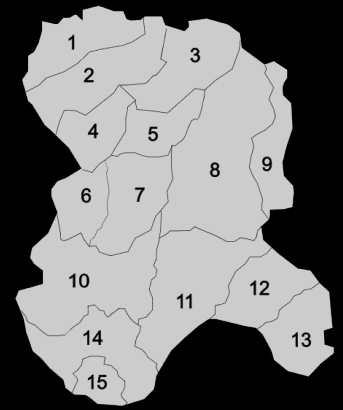
- Artificial data based on info from Trafikverket



Experiments

Base scenario:

- Artificial data based on info from Trafikverket
- Task load based on train movements

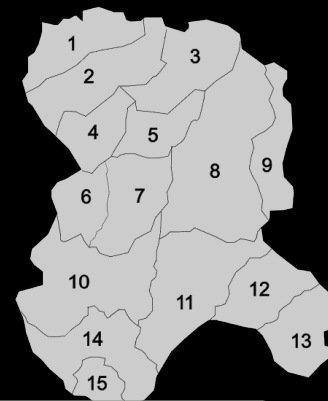


Type and (list of areas)	Night time (0-6)	Morning rush (6-9)	Evening rush (15-20)	Day time (9-15 & 20-24)
Single-track (1,10,11,12,13)	{2,3}	{14,15,16}	{14,15,16}	{9,10,11}
Double-track (2,5,6,8,9,14,15)	{9,10,11}	{9,10,11}	{19,20,21}	{9,10,11}
Complex (3,4,7)	{10,11,12,13,14}	{13,14,15,16}	{14,15,16,17}	{10,11,12,13,14}

Experiments

Base scenario:

- Artificial data based on info from Trafikverket
- Task load based on train movements

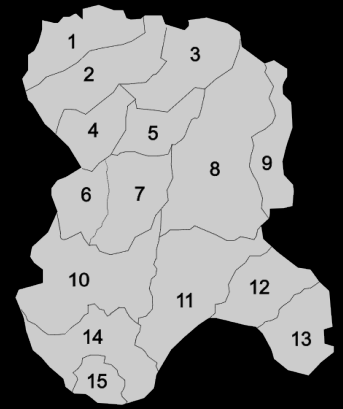


	p0	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13	p14	p15	p16	p17	p18	p19	p20	p21	p22	p23
ar.1	2	3	3	2	3	3	15	15	16	9	9	11	10	11	11	9	14	16	14	10	10	10	10	9
ar.2	11	11	10	11	10	9	11	11	9	11	9	9	11	9	11	21	21	21	19	20	11	11	9	9
ar.3	11	12	13	11	11	11	15	14	15	13	12	13	13	11	10	13	16	16	16	11	13	14	10	12
ar.4	11	13	13	13	10	11	13	16	14	13	13	13	14	11	13	12	16	15	15	13	10	12	13	10
ar.5	11	11	11	10	9	9	9	9	11	11	9	9	9	10	11	19	20	21	19	20	9	9	10	9
ar.6	9	9	10	10	11	10	9	9	10	11	10	10	10	9	9	21	20	20	20	19	9	10	11	10
ar.7	11	14	11	13	13	12	16	15	14	10	13	12	12	13	14	14	14	15	16	12	11	11	11	13
ar.8	9	11	11	10	10	10	11	9	9	10	9	11	11	9	11	20	19	19	20	21	11	11	10	10
ar.9	10	10	9	9	11	10	10	11	9	10	10	10	10	10	11	19	19	19	21	21	11	11	9	10
ar.10	3	2	3	2	3	3	16	14	16	11	9	10	10	10	9	11	14	15	15	9	11	11	9	11
ar.11	2	2	2	3	3	2	14	14	15	11	9	10	10	10	10	11	14	15	16	11	11	9	11	10
ar.12	2	3	2	2	3	2	16	14	14	9	11	9	11	10	10	10	14	14	14	9	10	10	10	9
ar.13	3	3	2	2	3	2	14	16	14	10	9	10	10	9	11	11	15	16	16	11	11	10	11	11
ar.14	9	11	11	9	10	10	9	9	9	11	10	11	10	11	9	21	20	20	20	20	11	9	10	10
ar.15	9	9	10	9	11	11	10	10	10	11	10	10	9	10	11	20	19	19	19	19	10	10	9	9

Experiments

Base scenario:

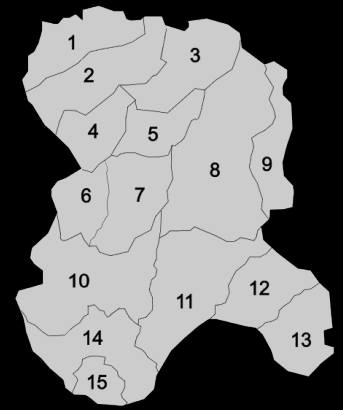
- Artificial data based on info from Trafikverket
- Task load based on train movements
- 22 dispatchers (available) to cover 15 areas



Experiments

Base scenario:

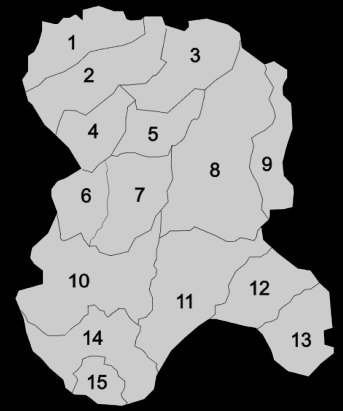
- Artificial data based on info from Trafikverket
- Task load based on train movements
- 22 dispatchers (available) to cover 15 areas
- Min and max shift length: [4h-11h]



Experiments

Base scenario:

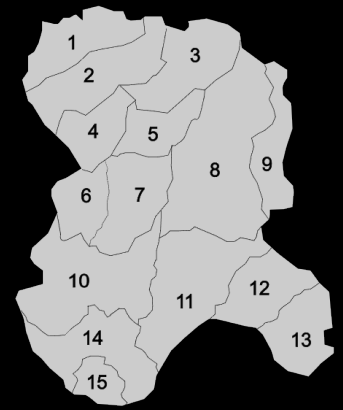
- Artificial data based on info from Trafikverket
- Task load based on train movements
- 22 dispatchers (available) to cover 15 areas
- Min and max shift length: [4h-11h]
- Min rest between shifts: 11h



Experiments

Base scenario:

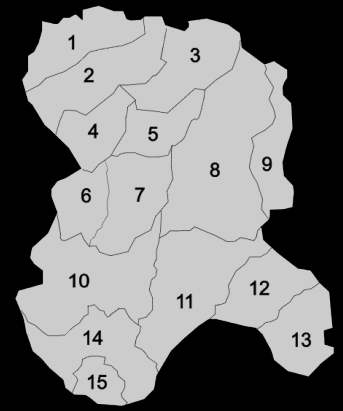
- Artificial data based on info from Trafikverket
- Task load based on train movements
- 22 dispatchers (available) to cover 15 areas
- Min and max shift length: [4h-11h]
- Min rest between shifts: 11h
- Max task load: 30 movements



Experiments

Base scenario:

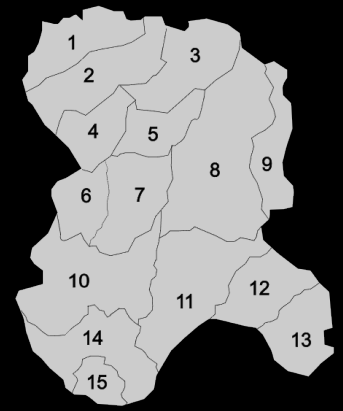
- Artificial data based on info from Trafikverket
- Task load based on train movements
- 22 dispatchers (available) to cover 15 areas
- Min and max shift length: [4h-11h]
- Min rest between shifts: 11h
- Max task load: 30 movements
- Max size of area combinations: 3



Experiments

Base scenario:

- Artificial data based on info from Trafikverket
- Task load based on train movements
- 22 dispatchers (available) to cover 15 areas
- Min and max shift length: [4h-11h]
- Min rest between shifts: 11h
- Max task load: 30 movements
- Max size of area combinations: 3
- Endorsement ratio: 100%



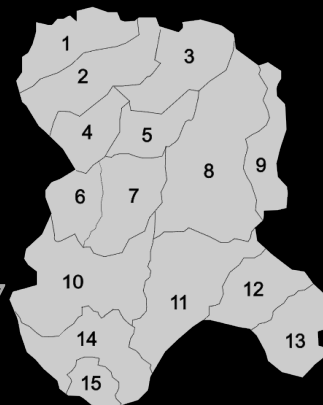
Result Base Scenario



	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
D1												5;8;9	6;1	11;12;13	2;3	1;2	12;13	9	15	3;4	6	11;12;13		
D2	15	10	4;6	3	11;14;15	5;8;9	6;7	6;7	11;12	11;12;13	8;9													
D3														4	4;5	8	14	4;7	9	12;13	12;13	3	3;5	13
D4		3;5	1;2;3	11;12;13	12;13	1;2;3	1;2	1;2	3;4	1;2	1;2;3													
D5				10;14;15	1;2;3	4;6	10;11	10;11	14;15	6	11;12;13	10;14	1;2	15										
D6	5	4														7	5	14	5	5	7;1	6	7	1;2;3
D7	3	1	9	5	5;8;9	7	12;13	3;4														10	6;8;10	12
D8						11;12;13	14;15	12;13	1;2	14;15	14	1;2	8;9	1	1									
D9						10;14	5;8;9	14;15	5;8;9	7;8;9	6;7	3;4	4;7	8;9	6;8;10	12;13								
D10									6	10	10	6;7	12;13	2;3	11;14;15	14	4;7	8	11					
D11	6;10;14	6;7	11;12;13	1;2;4															12;13	14	3;4	1;2	14;15	10
D12	12	14;15	5;7;10														3	10;11	6	15	11	4	13	8;11
D13																15	2	15	3	9	5;8	8	1;2	7
D14	1;2;4	2														5	6	1	10	8	9	7	11;12	4;5;6
D15									13	4;5	4;5	11;12;13	5	5	7	9	15	6	14					
D16	13															3;4	1	5	2	10	15	14;15	9	9
D17	7	11;12;13	8	6;7	4;6	15	3;4	5;8;9	7;1	3	15													
D18												15	11;14;15	7	12;13	10;11	8	3	8	1;2	14	9		
D19																	9	12;13	7	11				
D20													3	6;10;14	9	6	10;11	2	4	7				
D21	8;9;11	8;9	14;15	8;9	7;1														1	6	1;2	5	4	14;15

Result Base Scenario

- Used 21 dispatchers
- Min shift length: 4h
- Max shift length: 11h
- Avg. shift length: 10.23h
- Avg. nr. Assigned areas: 1.67
- Run time: 57s



	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
D1												5;8;9	6;1	11;12;13	2;3	1;2	12;13	9	15	3;4	6	11;12;13		
D2	15	10	4;6	3	11;14;15	5;8;9	6;7	6;7	11;12	11;12;13	8;9													
D3														4	4;5	8	14	4;7	9	12;13	12;13	3	3;5	13
D4		3;5	1;2;3	11;12;13	12;13	1;2;3	1;2	1;2	3;4	1;2	1;2;3													
D5				10;14;15	1;2;3	4;6	10;11	10;11	14;15	6	11;12;13	10;14	1;2	15										
D6	5	4														7	5	14	5	5	7;1	6	7	1;2;3
D7	3	1	9	5	5;8;9	7	12;13	3;4														10	6;8;10	12
D8						11;12;13	14;15	12;13	1;2	14;15	14	1;2	8;9	1	1									
D9						10;14	5;8;9	14;15	5;8;9	7;8;9	6;7	3;4	4;7	8;9	6;8;10	12;13								
D10									6	10	10	6;7	12;13	2;3	11;14;15	14	4;7	8	11					
D11	6;10;14	6;7	11;12;13	1;2;4															12;13	14	3;4	1;2	14;15	10
D12	12	14;15	5;7;10														3	10;11	6	15	11	4	13	8;11
D13																15	2	15	3	9	5;8	8	1;2	7
D14	1;2;4	2													5	6	1	10	8	9	7	11;12	4;5;6	
D15									13	4;5	4;5	11;12;13	5	5	7	9	15	6	14					
D16	13															3;4	1	5	2	10	15	14;15	9	9
D17	7	11;12;13	8	6;7	4;6	15	3;4	5;8;9	7;1	3	15													
D18												15	11;14;15	7	12;13	10;11	8	3	8	1;2	14	9		
D19																	9	12;13	7	11				
D20													3	6;10;14	9	6	10;11	2	4	7				
D21	8;9;11	8;9	14;15	8;9	7;1														1	6	1;2	5	4	14;15

Changing the Ratio of Endorsements Hold

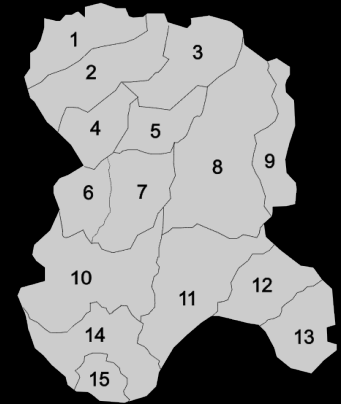
	Base scenario (100%)	$E_{1/2}$	$E_{1/3}$
nr dispatchers	21	21	22
min shift length	4h	9h	10h
max shift length	11h	11h	11h
avg. shift length	10.23h	10.86h	10.95h
avg. nr. assigned areas	1.67	1.67	1.49
run time	57s	20s	19s

Different Maximum Cardinality of Connected Components

	Base scenario (3 areas)	M_4	M_2	M_1
nr dispatchers	21	21	22	33
min shift length	4h	5h	4h	10h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.09h	10.57h	10.91h
avg. nr. assigned areas	1.67	1.7	1.55	1
run time	57s	97s	39s	29s

Removing Areas with High Degree in Adjacency Graph

	Base scenario (15 areas)	A_{10}	$A_{7,10}$	$A_{5,7,10}$
nr dispatchers	21	20	19	18
min shift length	4h	8h	10h	11h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.05h	10.95h	11h
avg. nr. assigned areas	1.67	1.6	1.5	1.45
run time	57s	39s	25s	21s



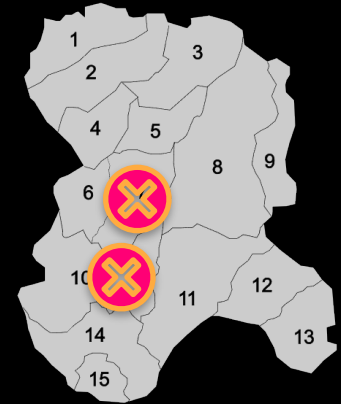
Removing Areas with High Degree in Adjacency Graph

	Base scenario (15 areas)	A_{10}	$A_{7,10}$	$A_{5,7,10}$
nr dispatchers	21	20	19	18
min shift length	4h	8h	10h	11h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.05h	10.95h	11h
avg. nr. assigned areas	1.67	1.6	1.5	1.45
run time	57s	39s	25s	21s



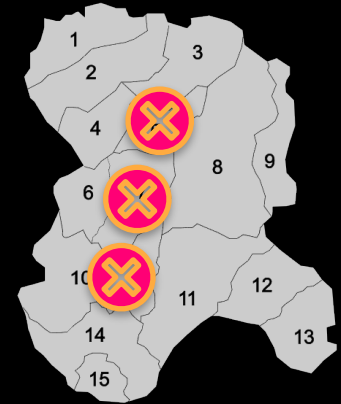
Removing Areas with High Degree in Adjacency Graph

	Base scenario (15 areas)	A_{10}	$A_{7,10}$	$A_{5,7,10}$
nr dispatchers	21	20	19	18
min shift length	4h	8h	10h	11h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.05h	10.95h	11h
avg. nr. assigned areas	1.67	1.6	1.5	1.45
run time	57s	39s	25s	21s



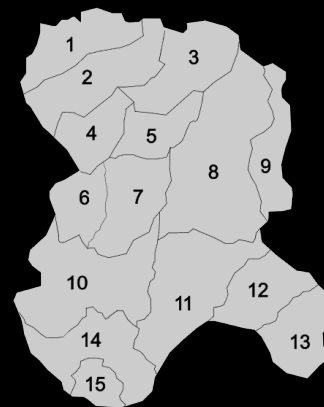
Removing Areas with High Degree in Adjacency Graph

	Base scenario (15 areas)	A_{10}	$A_{7,10}$	$A_{5,7,10}$
nr dispatchers	21	20	19	18
min shift length	4h	8h	10h	11h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.05h	10.95h	11h
avg. nr. assigned areas	1.67	1.6	1.5	1.45
run time	57s	39s	25s	21s



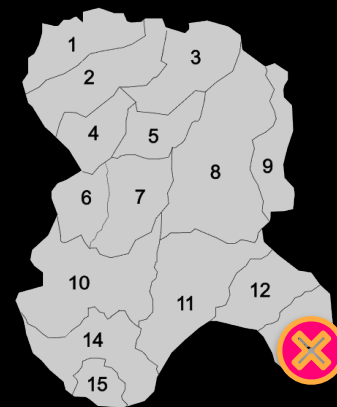
Removing Areas with Low Degree in Adjacency Graph

	Base scenario (15 areas)	A_{13}	$A_{13,15}$	$A_{13,14,15}$
nr dispatchers	21	20	18	17
min shift length	4h	8h	5h	5h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.55h	10.05h	10.58h
avg. nr. assigned areas	1.67	1.59	1.72	1.6
run time	57s	305s	245s	252s



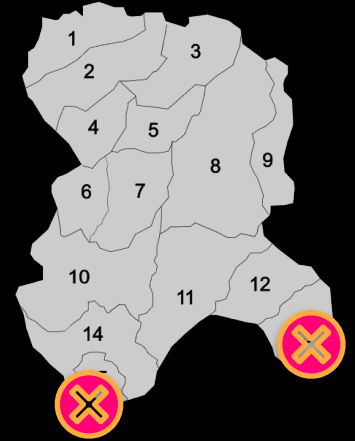
Removing Areas with Low Degree in Adjacency Graph

	Base scenario (15 areas)	A_{13}	$A_{13,15}$	$A_{13,14,15}$
nr dispatchers	21	20	18	17
min shift length	4h	8h	5h	5h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.55h	10.05h	10.58h
avg. nr. assigned areas	1.67	1.59	1.72	1.6
run time	57s	305s	245s	252s



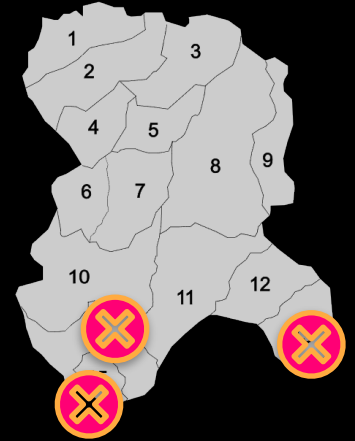
Removing Areas with Low Degree in Adjacency Graph

	Base scenario (15 areas)	A_{13}	$A_{13,15}$	$A_{13,14,15}$
nr dispatchers	21	20	18	17
min shift length	4h	8h	5h	5h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.55h	10.05h	10.58h
avg. nr. assigned areas	1.67	1.59	1.72	1.6
run time	57s	305s	245s	252s



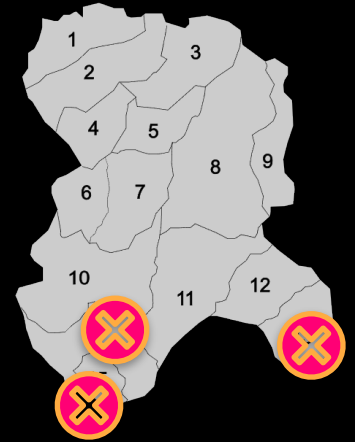
Removing Areas with Low Degree in Adjacency Graph

	Base scenario (15 areas)	A_{13}	$A_{13,15}$	$A_{13,14,15}$
nr dispatchers	21	20	18	17
min shift length	4h	8h	5h	5h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.55h	10.05h	10.58h
avg. nr. assigned areas	1.67	1.59	1.72	1.6
run time	57s	305s	245s	252s



Removing Areas with Low Degree in Adjacency Graph

	Base scenario (15 areas)	A_{13}	$A_{13,15}$	$A_{13,14,15}$
nr dispatchers	21	20	18	17
min shift length	4h	8h	5h	5h
max shift length	11h	11h	11h	11h
avg. shift length	10.23h	10.55h	10.05h	10.58h
avg. nr. assigned areas	1.67	1.59	1.72	1.6
run time	57s	305s	245s	252s



Runtime does not necessarily decrease with fewer areas

What Should We Improve?

	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
D1												5;8;9	6;1	11;12;13	2;3	1;2	12;13	9	15	3;4	6	11;12;13		
D2	15	10	4;6	3	11;14;15	5;8;9	6;7	6;7	11;12	11;12;13	8;9													
D3														4	4;5	8	14	4;7	9	12;13	12;13	3	3;5	13
D4		3;5	1;2;3	11;12;13	12;13	1;2;3	1;2	1;2	3;4	1;2	1;2;3													
D5				10;14;15	1;2;3	4;6	10;11	10;11	14;15	6	11;12;13	10;14	1;2	15										
D6	5	4														7	5	14	5	5	7;1	6	7	1;2;3
D7	3	1	9	5	5;8;9	7	12;13	3;4														10	6;8;10	12
D8						11;12;13	14;15	12;13	1;2	14;15	14	1;2	8;9	1	1									
D9						10;14	5;8;9	14;15	5;8;9	7;8;9	6;7	3;4	4;7	8;9	6;8;10	12;13								
D10									6	10	10	6;7	12;13	2;3	11;14;15	14	4;7	8	11					
D11	6;10;14	6;7	11;12;13	1;2;4														12;13	14	3;4	1;2	14;15	10	
D12	12	14;15	5;7;10														3	10;11	6	15	11	4	13	8;11
D13																15	2	15	3	9	5;8	8	1;2	7
D14	1;2;4	2														5	6	1	10	8	9	7	11;12	4;5;6
D15									13	4;5	4;5	11;12;13	5	5	7	9	15	6	14					
D16	13															3;4	1	5	2	10	15	14;15	9	9
D17	7	11;12;13	8	6;7	4;6	15	3;4	5;8;9	7;1	3	15													
D18												15	11;14;15	7	12;13	10;11	8	3	8	1;2	14	9		
D19																	9	12;13	7	11				
D20													3	6;10;14	9	6	10;11	2	4	7				
D21	8;9;11	8;9	14;15	8;9	7;1														1	6	1;2	5	4	14;15

What Should We Improve?

- Runtime

	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
D1												5;8;9	6;1	11;12;13	2;3	1;2	12;13	9	15	3;4	6	11;12;13		
D2	15	10	4;6	3	11;14;15	5;8;9	6;7	6;7	11;12	11;12;13	8;9													
D3														4	4;5	8	14	4;7	9	12;13	12;13	3	3;5	13
D4		3;5	1;2;3	11;12;13	12;13	1;2;3	1;2	1;2	3;4	1;2	1;2;3													
D5				10;14;15	1;2;3	4;6	10;11	10;11	14;15	6	11;12;13	10;14	1;2	15										
D6	5	4														7	5	14	5	5	7;1	6	7	1;2;3
D7	3	1	9	5	5;8;9	7	12;13	3;4														10	6;8;10	12
D8						11;12;13	14;15	12;13	1;2	14;15	14	1;2	8;9	1	1									
D9						10;14	5;8;9	14;15	5;8;9	7;8;9	6;7	3;4	4;7	8;9	6;8;10	12;13								
D10									6	10	10	6;7	12;13	2;3	11;14;15	14	4;7	8	11					
D11	6;10;14	6;7	11;12;13	1;2;4														12;13	14	3;4	1;2	14;15	10	
D12	12	14;15	5;7;10														3	10;11	6	15	11	4	13	8;11
D13																15	2	15	3	9	5;8	8	1;2	7
D14	1;2;4	2														5	6	1	10	8	9	7	11;12	4;5;6
D15									13	4;5	4;5	11;12;13	5	5	7	9	15	6	14					
D16	13															3;4	1	5	2	10	15	14;15	9	9
D17	7	11;12;13	8	6;7	4;6	15	3;4	5;8;9	7;1	3	15													
D18												15	11;14;15	7	12;13	10;11	8	3	8	1;2	14	9		
D19																	9	12;13	7	11				
D20													3	6;10;14	9	6	10;11	2	4	7				
D21	8;9;11	8;9	14;15	8;9	7;1														1	6	1;2	5	4	14;15

What Should We Improve?

- Runtime
- Too short shifts

	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
D1												5;8;9	6;1	11;12;13	2;3	1;2	12;13	9	15	3;4	6	11;12;13		
D2	15	10	4;6	3	11;14;15	5;8;9	6;7	6;7	11;12	11;12;13	8;9													
D3														4	4;5	8	14	4;7	9	12;13	12;13	3	3;5	13
D4		3;5	1;2;3	11;12;13	12;13	1;2;3	1;2	1;2	3;4	1;2	1;2;3													
D5				10;14;15	1;2;3	4;6	10;11	10;11	14;15	6	11;12;13	10;14	1;2	15										
D6	5	4														7	5	14	5	5	7;1	6	7	1;2;3
D7	3	1	9	5	5;8;9	7	12;13	3;4														10	6;8;10	12
D8						11;12;13	14;15	12;13	1;2	14;15	14	1;2	8;9	1	1									
D9						10;14	5;8;9	14;15	5;8;9	7;8;9	6;7	3;4	4;7	8;9	6;8;10	12;13								
D10									6	10	10	6;7	12;13	2;3	11;14;15	14	4;7	8	11					
D11	6;10;14	6;7	11;12;13	1;2;4														12;13	14	3;4	1;2	14;15	10	
D12	12	14;15	5;7;10														3	10;11	6	15	11	4	13	8;11
D13																15	2	15	3	9	5;8	8	1;2	7
D14	1;2;4	2														5	6	1	10	8	9	7	11;12	4;5;6
D15									13	4;5	4;5	11;12;13	5	5	7	9	15	6	14					
D16	13															3;4	1	5	2	10	15	14;15	9	9
D17	7	11;12;13	8	6;7	4;6	15	3;4	5;8;9	7;1	3	15													
D18												15	11;14;15	7	12;13	10;11					14	9		
D19																								
D20													3	6;10;14	9	6	9	12;13	7	11				
D21	8;9;11	8;9	14;15	8;9	7;1														1	6	1;2	5	4	14;15

What Should We Improve?

- Runtime
- Too short shifts
- Undesirable starting times

	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
D1												5;8;9	6;1	11;12;13	2;3	1;2	12;13	9	15	3;4	6	11;12;13			
D2	15	10	4;6	3	11;14;15	5;8;9	6;7	6;7	11;12	11;12;13	8;9														
D3														4	4;5	8	14	4;7	9	12;13	12;13	3	3;5	13	
D4		3;5	1;2;3	11;12;13	12;13	1;2;3	1;2	1;2	3;4	1;2	1;2;3														
D5				10;14;15	1;2;3	4;6	10;11	10;11	14;15	6	11;12;13	10;14	1;2	15											
D6	5	4														7	5	14	5	5	7;1	6	7	1;2;3	
D7	3	1	9	5	5;8;9	7	12;13	3;4														10	6;8;10	12	
D8						11;12;13	14;15	12;13	1;2	14;15	14	1;2	8;9	1	1										
D9						10;14	5;8;9	14;15	5;8;9	7;8;9	6;7	3;4	4;7	8;9	6;8;10	12;13									
D10									6	10	10	6;7	12;13	2;3	11;14;15	14	4;7	8	11						
D11	6;10;14	6;7	11;12;13	1;2;4														12;13	14	3;4	1;2	14;15	10		
D12	12	14;15	5;7;10														3	10;11	6	15	11	4	13	8;11	
D13																15	2	15	3	9	5;8	8	1;2	7	
D14	1;2;4	2														5	6	1	10	8	9	7	11;12	4;5;6	
D15									13	4;5	4;5	11;12;13	5	5	7	9	15	6	14						
D16	13															3;4	1	5	2	10	15	14;15	9	9	
D17	7	11;12;13	8	6;7	4;6	15	3;4	5;8;9	7;1	3	15														
D18												15	11;14;15	7	12;13	10;11					14	9			
D19																									
D20													3	6;10;14	9	6	9	12;13	7	11					
D21	8;9;11	8;9	14;15	8;9	7;1															1	6	1;2	5	4	14;15

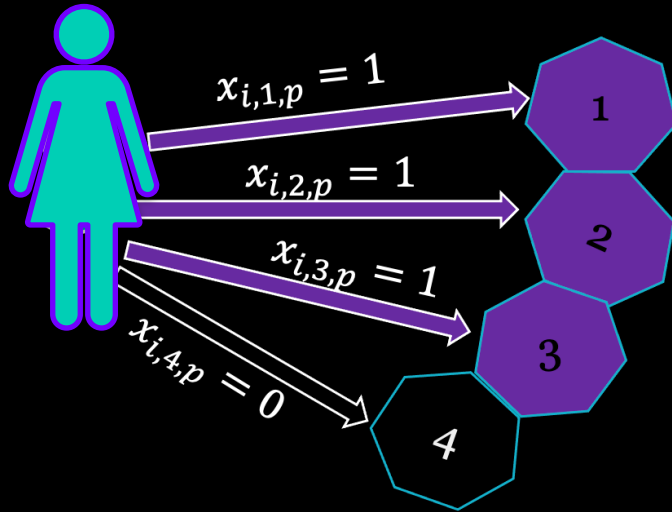
What Should We Improve?

- Runtime
- Too short shifts
- Undesirable starting times
- Too many handovers

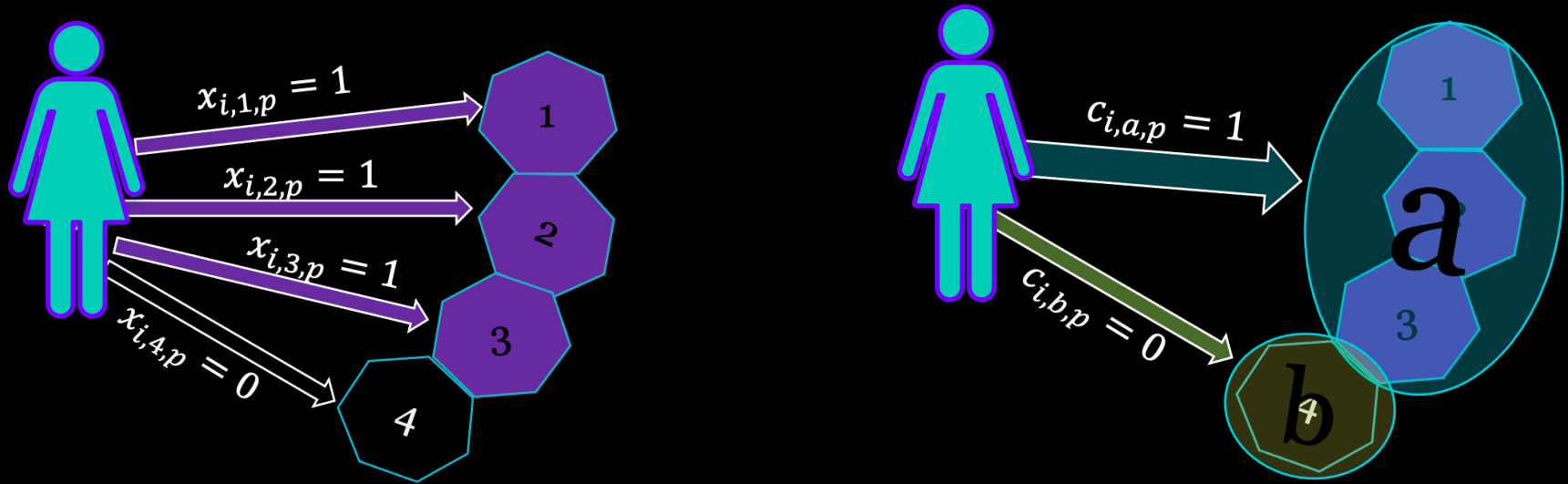
	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
D1												5;8;9	6;1	11;12;13	2;3	1;2	12;13	9	15	3;4	6	11;12;13		
D2	15	10	4;6	3	11;14;15	5;8;9	6;7	6;7	11;12	11;12;13	8;9													
D3														4	4;5	8	14	4;7	9	12;13	12;13	3	3;5	13
D4		3;5	1;2;3	11;12;13	12;13	1;2;3	1;2	1;2	3;4	1;2	1;2;3													
D5				10;14;15	1;2;3	4;6	10;11	10;11	14;15	6	11;12;13	10;14	1;2	15										
D6	5	4														7	5	14	5	5	7;1	6	7	1;2;3
D7	3	1	9	5	5;8;9	7	12;13	3;4													10	6;8;10	12	
D8									1;2	14;15	14	1;2	8;9	1	1									
D9						10;14	5;8;9	14;15	5;8;9	7;8;9	6;7	3;4	4;7	8;9	6;8;10	12;13								
D10									6	10	10	6;7	12;13	2;3	11;14;15	14	4;7	8	11					
D11	6;10;14	6;7	11;12;13	1;2;4														12;13	14	3;4	1;2	14;15	10	
D12	12	14;15	5;7;10														3	10;11	6	15	11	4	13	8;11
D13																15	2	15	3	9	5;8	8	1;2	7
D14	1;2;4	2														5	6	1	10	8	9	7	11;12	4;5;6
D15									13	4;5	4;5	11;12;13	5	5	7	9	15	6	14					
D16	13															3;4	1	5	2	10	15	14;15	9	9
D17	7	11;12;13	8	6;7	4;6	15	3;4	5;8;9	7;1	3	15													
D18												15	11;14;15	7	12;13	10;11					14	9		
D19																								
D20													3	6;10;14	9	6	9	12;13	7	11				
D21	8;9;11	8;9	14;15	8;9	7;1														1	6	1;2	5	4	14;15

Runtime: Stronger Formulation

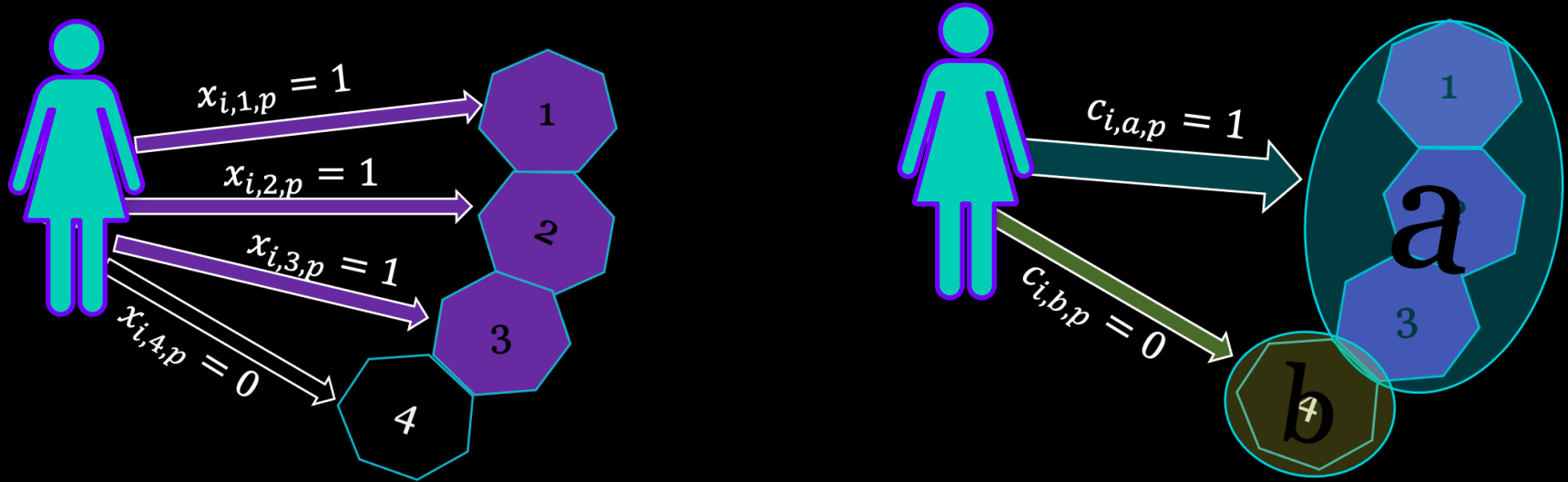
Runtime: Stronger Formulation



Runtime: Stronger Formulation

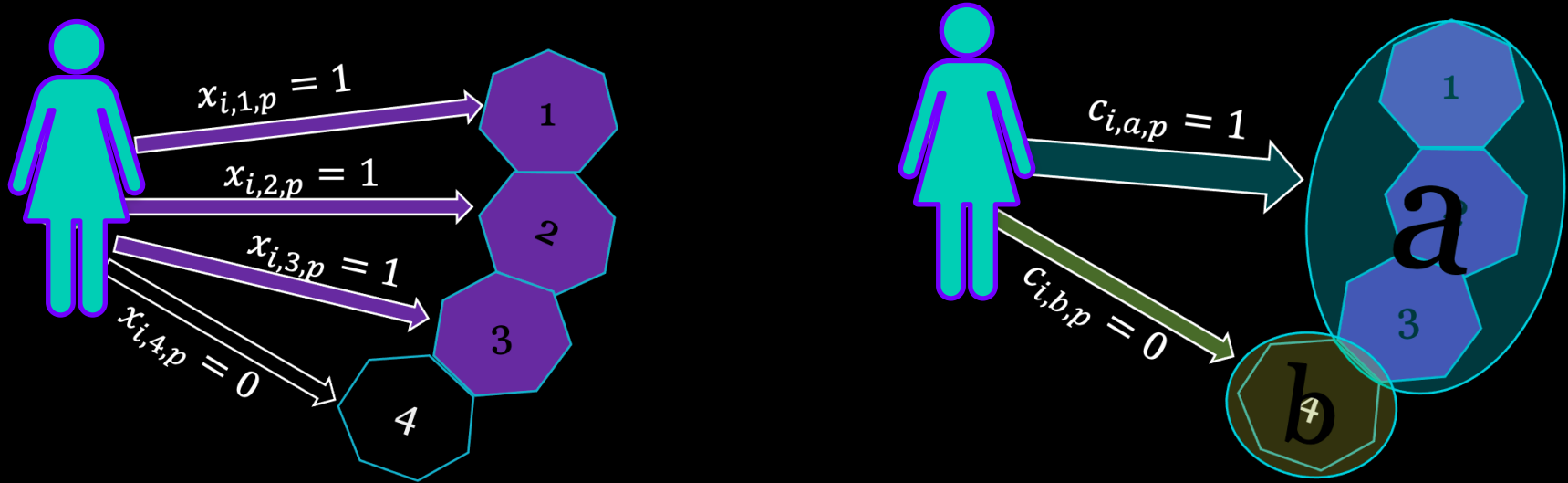


Runtime: Stronger Formulation



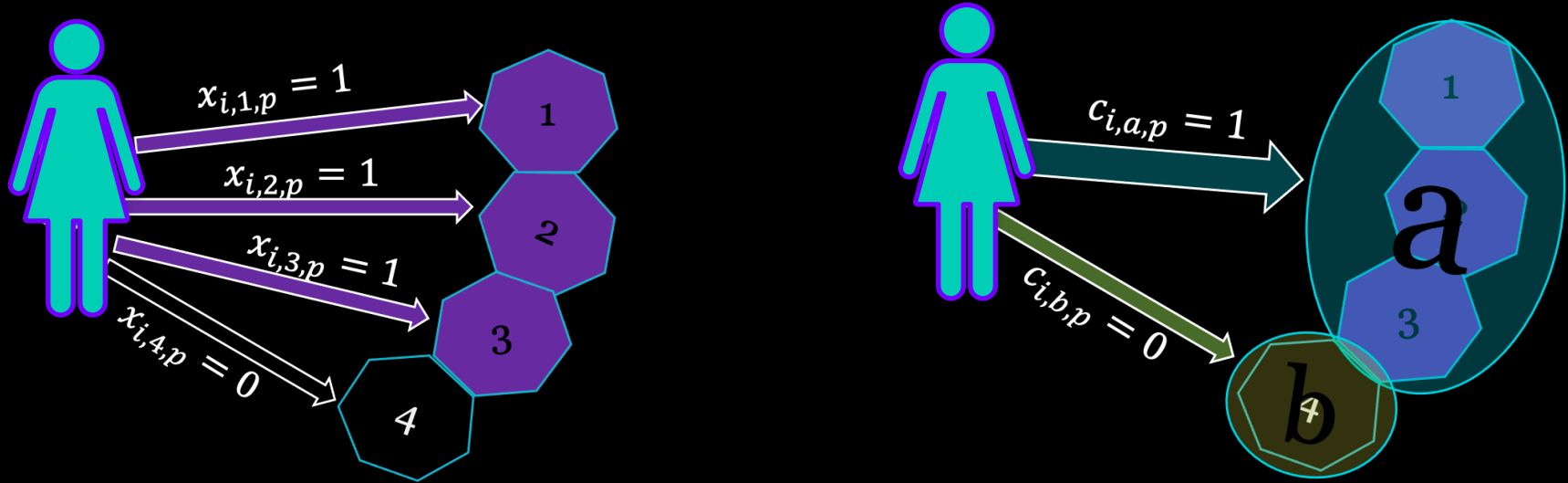
- Assignments steered by variables $c_{i,l,k}$ (CCs) instead of $x_{i,j,k}$

Runtime: Stronger Formulation



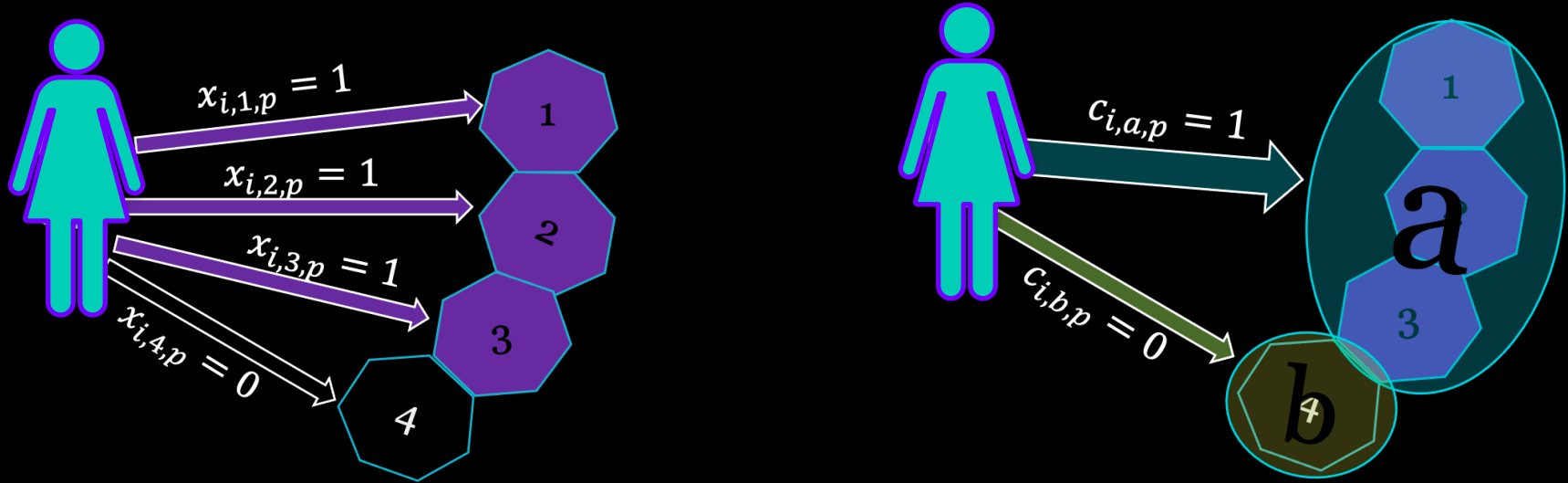
- Assignments steered by variables $c_{i,l,k}$ (CCs) instead of $x_{i,j,k}$
- Connect variables $x_{i,j,k}$ and $c_{i,l,k}$ to track area assignments

Runtime: Stronger Formulation



- Assignments steered by variables $c_{i,l,k}$ (CCs) instead of $x_{i,j,k}$
- Connect variables $x_{i,j,k}$ and $c_{i,l,k}$ to track area assignments
- Runtime reduced by about 90%

Runtime: Stronger Formulation



- Assignments steered by variables $c_{i,l,k}$ (CCs) instead of $x_{i,j,k}$
- Connect variables $x_{i,j,k}$ and $c_{i,l,k}$ to track area assignments
- Runtime reduced by about 90%

Old Model

$$\sum_{j \in A} x_{i,j,k} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$x_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq x_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} \geq c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

New Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC
- If i works during k, they must be assigned to one CC
- Negative connection of x and c

Old Model

$$\sum_{j \in A} T_{i,j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$T_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq \sum_{j \in A} x_{i,j,k} - 1 \quad \forall i \in D, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} T_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} > c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = 1 \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall \ell \in C, \forall j \in A \setminus \{\ell\}, \forall k \in P \quad (15)$$

New Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC
- If i works during k, they must be assigned to one CC
- Negative connection of x and c

Old Model

$$\sum_{j \in A} T_{i,j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$T_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} T_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} > c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = 1 \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in A \setminus \{\ell\} \quad (15)$$

New Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC
- If i works during k, they must be assigned to one CC
- Negative connection of x and c

Old Model

$$\sum_{j \in A} T_{i,j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$T_{i,j,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \geq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+R^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} T_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} > c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = 1 \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in A \setminus \{\ell\} \quad (15)$$

New Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC
- If i works during k, they must be assigned to one CC
- Negative connection of x and c

$a_{\ell,j}$ is equal to 1 whenever area j is an element of area combination ℓ .

Old Model

$$\sum_{j \in A} TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (1)$$

$$v_{i,k} \leq e_{i,j} \quad \forall i \in D, \forall j \in A, \forall k \in P \quad (2)$$

$$v_{i,k} \geq y_{i,k} - y_{i,(k-1) \pmod p} \quad \forall i \in D, \forall k \in P \quad (3)$$

$$v_{i,k} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (4)$$

$$\sum_{\mu=k+1-T^{\min}}^k v_{i,\mu \pmod p} \leq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (5)$$

$$\sum_{\mu=k+1-T^{\max}}^k v_{i,\mu \pmod p} \geq y_{i,k} \quad \forall i \in D, \forall k \in P \quad (6)$$

$$v_{i,k} \leq q_i \quad \forall i \in D, \forall k \in P \quad (7)$$

$$\sum_{k \in P} v_{i,k} \geq q_i \quad \forall i \in D \quad (8)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (9)$$

$$y_{i,k} \leq \sum_{j \in A} x_{i,j,k} \quad \forall i \in D, \forall k \in P \quad (10)$$

$$\sum_{\mu=k+1}^{k+P^{\min}} v_{i,\mu \pmod p} \leq q_i - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (11)$$

$$\sum_{i \in D} x_{i,j,k} = 1 \quad \forall j \in A, k \in P \quad (12)$$

$$x_{i,j,k} > c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in \ell \quad (13)$$

$$\sum_{\ell \in C} c_{i,\ell,k} = 1 - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (14)$$

$$x_{i,j,k} \leq 1 - c_{i,\ell,k} \quad \forall i \in D, \forall k \in P, \forall \ell \in C, \forall j \in A \setminus \{\ell\} \quad (15)$$

New Model

- Taskload does not exceed the maximum
- Only assigned if dispatcher is qualified for the area
- Only start if at work, and was not at work before
- Minimum and maximum shift length
- If dispatcher starts, he works, and if he works, he must start
- If dispatcher i works during k, he must be assigned to some area j
- Minimum rest between shifts
- Each area must be assigned to exactly one dispatcher during each period
- If connected component ℓ is assigned to dispatcher i during k, so have to be all areas j in that CC
- If i works during k, they must be assigned to one CC
- Negative connection of x and c

$a_{\ell,j}$ is equal to 1 whenever area j is an element of area combination ℓ .

$$\sum_{j \in A} \sum_{\ell \in C} c_{i,\ell,k} \cdot a_{\ell,j} \cdot TL_{j,k} \leq TL^{\max} \quad \forall i \in D, \forall k \in P \quad (17)$$

$$c_{i,\ell,k} \leq e_{i,j} \quad \forall i \in D, \forall \ell \in C, \forall j \in \ell, \forall k \in P \quad (18)$$

$$\sum_{\ell \in C \setminus \{0\}} c_{i,\ell,k} = y_{i,k} \quad \forall i \in D, \forall k \in P \quad (19)$$

$$c_{i,0,k} = 1 - y_{i,k} \quad \forall i \in D, \forall k \in P \quad (20)$$

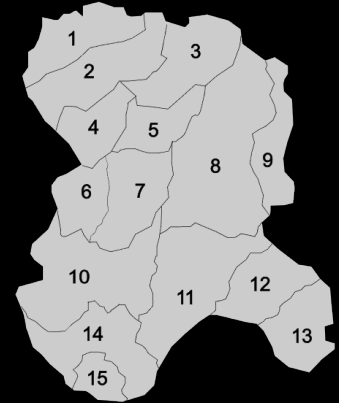
$$\sum_{\ell \in C \setminus \{0\}} \sum_{i \in D} a_{\ell,j} \cdot c_{i,\ell,k} = 1 \quad \forall k \in P, \forall j \in A \quad (21)$$

Experiments: Stronger Formulation

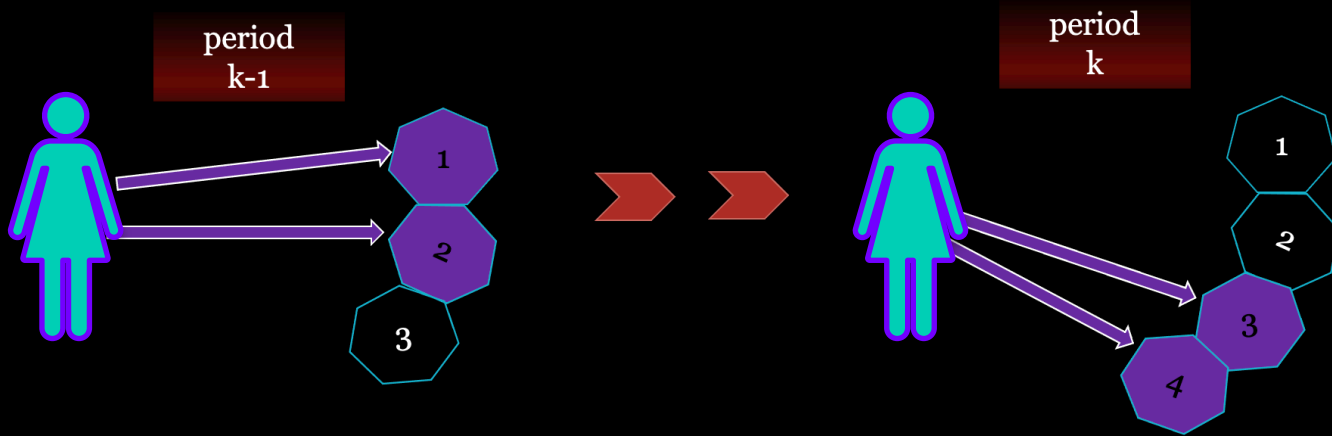
$I_{a,b,A}$

- a is the endorsement ratio
- b is the maximum size of area combinations
- Set A contains the areas that have been removed from the 15 areas in the basic scenario

In.	d	Basic model		Strong model		
		row; col	R(s)	row; col	R(s)	%dif
$I_{1,3,\emptyset}$	21	18221; 34120	58	3019; 27058	8.3	-86
$I_{1/2,3,\emptyset}$	21	9152; 10192	23.1	3015; 7605	8.4	-64
$I_{1/3,3,\emptyset}$	22	6038; 6127	21.6	3009; 4923	8.7	-60
$I_{1,4,\emptyset}$	21	37926; 38058	97	3019; 30622	18.4	-81
$I_{1,2,\emptyset}$	22	18221; 26662	39	3019; 19600	4.98	-87
$I_{1,1,\emptyset}$	33	4627; 14313	30.5	4592; 14313	69	+126
$I_{1,3,\{13\}}$	20	47985; 32272	305	2995; 25738	10.3	-97
$I_{1,3,\{13,15\}}$	18	45573; 31070	245	2971; 24264	9.36	-96
$I_{1,3,\{13,14,15\}}$	17	35155; 27190	244	2947; 21514	9.45	-96
$I_{1,3,\{10\}}$	20	17053; 27014	39	2995; 20524	7.9	-80
$I_{1,3,\{7,10\}}$	19	15929; 23890	25	2971; 17950	5.2	-79
$I_{1,3,\{5,7,10\}}$	18	15069; 20546	21	2947; 15024	4.64	-78



Handling Handovers–Approach #1: Minimize #Handovers

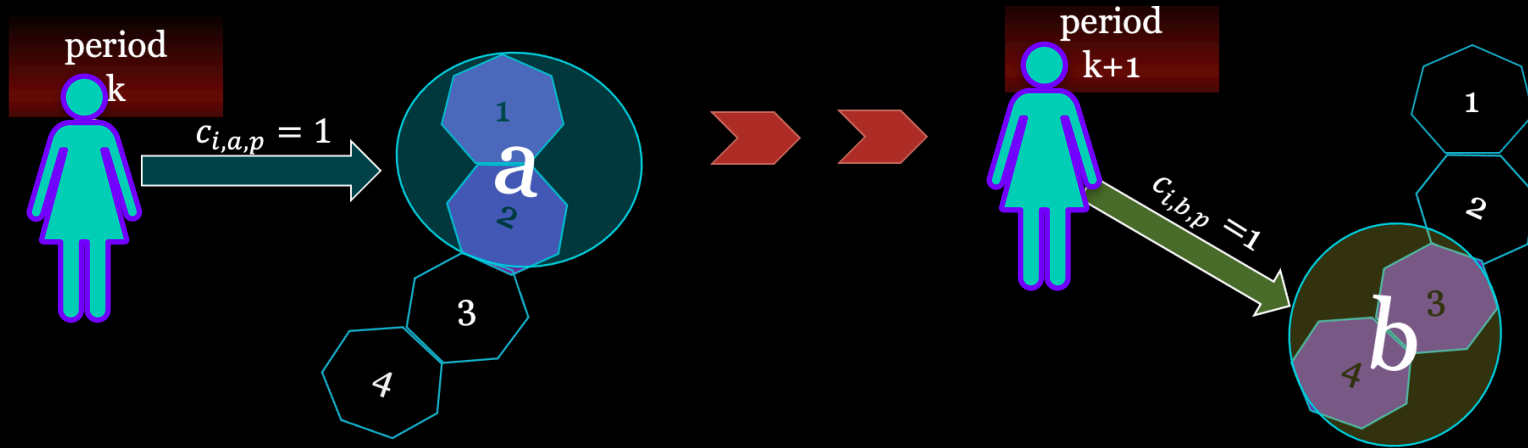


New constraints with binary variables $z_{i,j,k}=1$ if a dispatcher i gets a new area j in period k :

$$x_{i,j,k} - x_{i,j,(k-1) \bmod p} \leq z_{i,j,k} \quad \forall i \in D, \forall j \in A, \forall k \in P$$

New objective function: $\min . \sum_{i \in D} \sum_{j \in A} \sum_{k \in P} z_{i,j,k}$

Handling Handovers–Approach #2: Minimize #Handovers of CCs

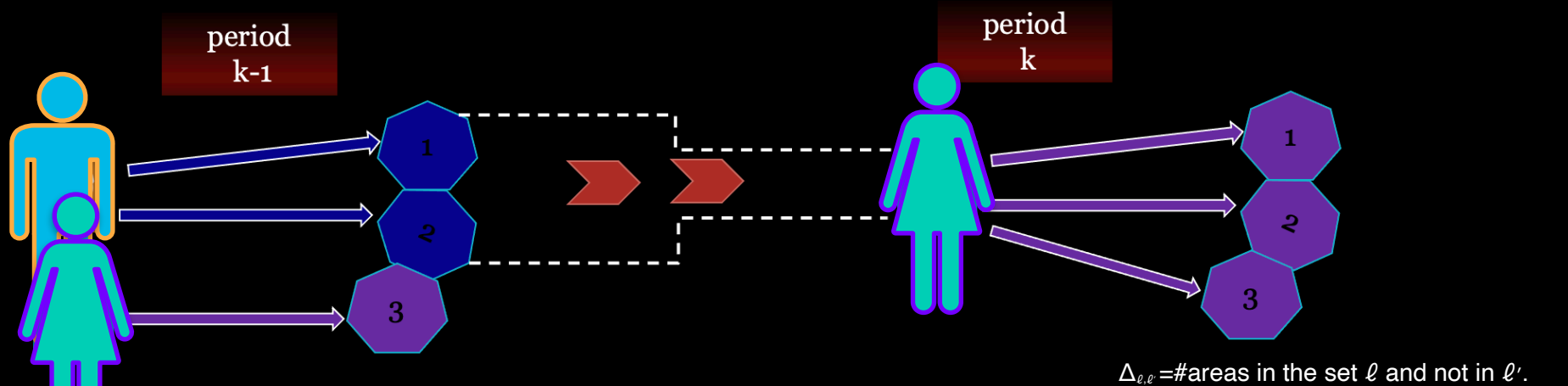


New constraints with new variables $h_{i,k}=1$ if dispatcher i is involved in a handover:

$$h_{i,k} \geq c_{i,\ell,k} - c_{i,\ell,(k-1) \bmod p} \quad \forall i \in D, \forall \ell \in C \setminus \{0\}, \forall k \in P$$

New objective: $\min . \sum_{i \in D} \sum_{k \in P} h_{i,k}$

Handling Handovers–Approach #3: Minimize #Newly Assigned Areas

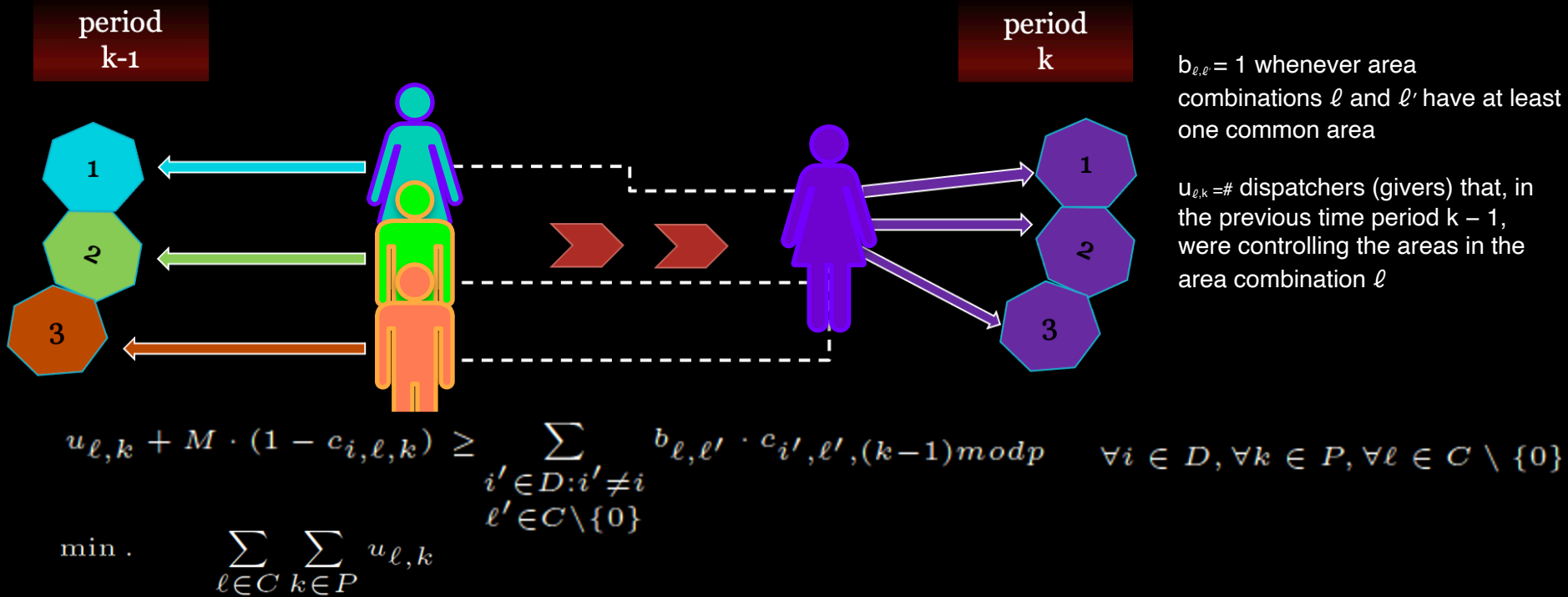


New constraints with variables $s_{i,k} = \text{\#new assigned areas to dispatcher } i \text{ in period } k$:

$$\Delta_{\ell, \ell'} \cdot (c_{i, \ell, k} + c_{i, \ell', (k-1) \bmod p} - 1) \leq s_{i, k} \quad \forall i \in D, \forall k \in P,$$

New objective: $\min . \sum_{i \in D} \sum_{k \in P} s_{i, k} \quad \forall \ell' \in C, \forall \ell \in C \setminus \{0\}$

Handling Handovers–Approach #4: Minimize #Givers



Experiments

Small instance with 8 areas and 5 periods (night, morning, noon, early evening, late evening), 8 used dispatcher:

	sm min. $\#disp$	Approach 1 min. $\sum z_{i,j,k}$	Approach 2 min. $\sum h_{i,k}$	Approach 3 min. $\sum s_{i,k}$	Approach 4 min. $\sum u_{l,k}$
#rows	239	1160	992	15648	1085
#columns	896	1488	928	928	994
runtime (s)	0.2	0.6	3.25	600	220
$\sum z_{i,j,k}$	31	16	18	16	16
$\sum h_{i,k}$	29	14	13	15	14
$\sum s_{i,k}$	31	16	18	16	16
$\sum u_{l,k}$	26	11	18	13	11

Experiments

Small instance with 8 areas and 5 periods (night, morning, noon, early evening, late evening), 8 used dispatcher:

	sm min. $\#disp$	Approach 1 min. $\sum z_{i,j,k}$	Approach 2 min. $\sum h_{i,k}$	Approach 3 min. $\sum s_{i,k}$	Approach 4 min. $\sum u_{l,k}$
#rows	239	1160	992	15648	1085
#columns	896	1488	928	928	994
runtime (s)	0.2	0.6	3.25	600	220
$\sum z_{i,j,k}$	31	16	18	16	16
$\sum h_{i,k}$	29	14	13	15	14
$\sum s_{i,k}$	31	16	18	16	16
$\sum u_{l,k}$	26	11	18	13	11

Experiments

Small instance with 8 areas and 5 periods (night, morning, noon, early evening, late evening), 8 used dispatcher:

	sm min. $\#disp$	Approach 1 min. $\sum z_{i,j,k}$	Approach 2 min. $\sum h_{i,k}$	Approach 3 min. $\sum s_{i,k}$	Approach 4 min. $\sum u_{l,k}$
#rows	239	1160	992	15648	1085
#columns	896	1488	928	928	994
runtime (s)	0.2	0.6	3.25	600	220
$\sum z_{i,j,k}$	31	16	18	16	16
$\sum h_{i,k}$	29	14	13	15	14
$\sum s_{i,k}$	31	16	18	16	16
$\sum u_{l,k}$	26	11	18	13	11

Experiments

Fixed the time resolution: 12 periods, 2h/period

#ar	d	Approach 1			Approach 2		
		row; col	R (s)	\sum $(z_{i,j,k}, h_{i,k}, s_{i,k}, u_{\ell,k})$	row; col	R (s)	\sum $(z_{i,j,k}, h_{i,k}, s_{i,k}, u_{\ell,k})$
8	11	3800; 5115	277	16,11,16,14	3440; 3245	15	16,11,16,14
10	13	5408; 8112	454	20,13,20,18	5255; 5304	49	20,13,20,20
12	16	7804; 12384	84	24,16,24,22	9324; 9930	282	24,16,24,22

Fixed #areas: 12 areas

#pr	d	Approach 1			Approach 2		
		row; col	R (s)	\sum $(z_{i,j,k}, h_{i,k}, s_{i,k}, u_{\ell,k})$	row; col	R (s)	\sum $(z_{i,j,k}, h_{i,k}, s_{i,k}, u_{\ell,k})$
5	11	2250; 3465	4.02	24,21,24,22	1963; 2288	58	29,19,29,22
8	15	4848; 7755	229	24,15,24,20	4551; 5130	18	24,15,24,20
12	16	7804; 12384	84	24,16,24,22	9324; 9930	282	24,16,24,22

Experiments: Real-World-Sized Instance

	pr.0	pr.1	pr.2	pr.3	pr.4	pr.5	pr.6	pr.7	pr.8	pr.9	pr.10	pr.11	pr.12	pr.13	pr.14	pr.15	pr.16	pr.17	pr.18	pr.19	pr.20	pr.21	pr.22	pr.23
D1																1					14			
D2										4		14		1	10	2	14	8	3	5	15	9		
D3	9	9	5							6	11	15	1	2	9	12	8	3	2	1				
D4										14	10	9	1	2	6	5	13	14	5	6	4			
D5	12		11		6	4		5															11	
D6	13	14	15	15	10	7	11	2	9	6												14	2	
D7						1	3	6	7	1	8	10	9	12	3	2								
D8						2	4	7	10	2	14	9	13	8	3	13								
D9						5	8	3		7														
D10						8	9	4	12	9	6	7	6	9										
D11																								
D12																								
D13	15	6	9		7	10	11	12																
D14																								
D15	3	10	7		8	9	15	14	12	13														
D16																								
D17																								
D18																								
D19																								
D20																								
D21																								
D22																								
D23																								

Approach 1: optimal solution in about 102 minutes
 Approach 2: optimality gap even after 72
 hours, but with good handover-measure values.

Experiments: Real-World-Sized Instance

	pr.0	pr.1	pr.2	pr.3	pr.4	pr.5	pr.6	pr.7	pr.8	pr.9	pr.10	pr.11	pr.12	pr.13	pr.14	pr.15	pr.16	pr.17	pr.18	pr.19	pr.20	pr.21	pr.22	pr.23
D1																1 2	14	8	3	5	14 15	9		
D2										4 6	11	15	1	1 2	9	12	8	3	2	1 2				
D3	9	9	5														3	9	8	15	1 2	3 8	12 13	8
D4									14 15	10 14	9	2	6	5	13	14	5	6	4					
D5	12 13	14	15	14	15	6 7 10	6 7 11	1 2	8 9	6													11 16 15	2
D6								1 2	3 4	6 7	7 10	2	8 10 14	9	8 13	12 13	3 8	2 3	13					
D7								5 8 9	4	11 12	8 9	6 7	5 6	8 9										
D8	15	6	9	7 10 11	12 13															7	18	8 9	4 7	6 9
D9	3	5 7 10	7	4 5	8 9	16 15	16 15	12 13														1 2	2	5 7
D10									3 4	15	1	11 12	13	11 12	13	1	15	6	2	11 12				
D11	1 2	12 13															9	4 7	12 13	13 12	10 11	11 12 13	3	3
D12	11																4	7	9	1	15	14	6 7	10 15
D13	10 14	4	6 8 10	13																10	6	5	5	1 4
D14	5	15	13	12	15	11 14	5 8	12 13	1	1													8 10	11 12
D15								2	15	15	10	10	2	9	15	8	2	5	14					
D16	4	8 11	3	5 8 9	3 5	4 7	10 11	14 15	9														9	14 15
D17											12 13	5 7	7	7	12	6	10 11	4 7	1 10	8 10				
D18											11 12	4 5	3 4	5 6	6 10	7 8	5	1 15	9 9					
D19												11 12	10 11	12 13	10 11	11 13	11	6 11	3 4	3 4	6	5 7	1	
D20												10 14 15	14 15	5 8	3 4	15	14	5		4 7	12 13			
D21	6 7 8	1 2 3	1 2 3	1 2 3	1 2 3	12 13	6 7	10 11	13 14	8														6 11

Approach 1: optimal solution in about 102 minutes
 Approach 2: optimality gap even after 72
 hours, but with good handover-measure values.

Experiments: Real-World-Sized Instance

	pr.0	pr.1	pr.2	pr.3	pr.4	pr.5	pr.6	pr.7	pr.8	pr.9	pr.10	pr.11	pr.12	pr.13	pr.14	pr.15	pr.16	pr.17	pr.18	pr.19	pr.20	pr.21	pr.22	pr.23
D1																								
D2																								
D3																								
D4																								
D5																								
D6																								
D7																								
D8																								
D9																								
D10																								
D11																								
D12																								
D13																								
D14																								
D15																								
D16																								
D17																								
D18																								
D19																								
D20																								
D21																								

	pr.0	pr.1	pr.2	pr.3	pr.4	pr.5	pr.6	pr.7	pr.8	pr.9	pr.10	pr.11	pr.12	pr.13	pr.14	pr.15	pr.16	pr.17	pr.18	pr.19	pr.20	pr.21	pr.22	pr.23
D1																								
D2																								
D3																								
D4																								
D5																								
D6																								
D7																								
D8																								
D9																								
D10																								
D11																								
D12																								
D13																								
D14																								
D15																								
D16																								
D17																								
D18																								
D19																								
D20																								
D21																								

Outlook: Current and Future Research

Outlook: Current and Future Research

- Expand the time horizon

Outlook: Current and Future Research

- Expand the time horizon
- New project (04/2024-09/2026) **Dispatching Areas: Combinations and Design (DACoD)**

Outlook: Current and Future Research

- Expand the time horizon
- New project (04/2024-09/2026) **Dispatching Areas: Combinations and Design (DACoD)**
- Which combinations of area qualifications are good?

Outlook: Current and Future Research

- Expand the time horizon
- New project (04/2024-09/2026) **Dispatching Areas: Combinations and Design (DACoD)**
- Which combinations of area qualifications are good?
- How to split the large area into smaller areas?

Outlook: Current and Future Research

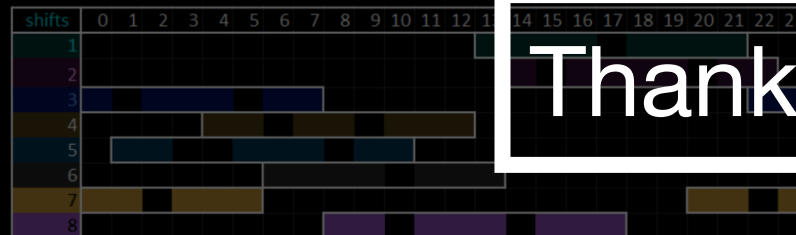
- Expand the time horizon
- New project (04/2024-09/2026) **Dispatching Areas: Combinations and Design (DACoD)**
- Which combinations of area qualifications are good?
- How to split the large area into smaller areas?
- Deduct and integrate workload thresholds into the framework

Outlook: Current and Future Research

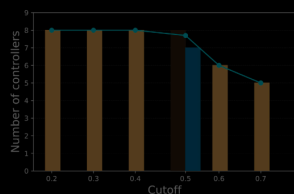
- Expand the time horizon
- New project (04/2024-09/2026) **Dispatching Areas: Combinations and Design (DACoD)**
- Which combinations of area qualifications are good?
- How to split the large area into smaller areas?
- Deduct and integrate workload thresholds into the framework
- Planning for en-route ATCOs (Area Control Centers and even Virtual Centers)

Outlook: Current and Future Research

- Expand the time horizon
- New project (04/2024-09/2026) **Dispatching Areas: Combinations and Design (DACoD)**
- Which combinations of area qualifications are good?
- How to split the large area into smaller areas?
- Deduct and integrate workload thresholds into the framework
- Planning for en-route ATCOs (Area Control Centers and even Virtual Centers)
- Continued study of ATCO workload



Thank you.

[illegible]

Assigning Airports to Remote Tower Modules—Constraints

$$\sum_{j \in A} period_{i,j,k} \leq RTM_{i,k} \cdot mA \quad \forall i \in R, \forall k \in P$$

$$\sum_{j \in A} mov_{i,j,k} \leq mMov \quad \forall i \in R, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \leq 1 \quad \forall j \in A, \forall k \in P$$

$$mov_{i,j,k} \leq period_{i,j,k} \cdot mMov \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} mov_{i,j,k} = Amov_{j,k} \quad \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \geq op_{j,k} \quad \forall j \in A, \forall k \in P$$

Assigning Airports to Remote Tower Modules—Constraints

1. Number of airports assigned to one module $\leq mA$

$$\sum_{j \in A} period_{i,j,k} \leq RTM_{i,k} \cdot mA \quad \forall i \in R, \forall k \in P$$

$$\sum_{j \in A} mov_{i,j,k} \leq mMov \quad \forall i \in R, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \leq 1 \quad \forall j \in A, \forall k \in P$$

$$mov_{i,j,k} \leq period_{i,j,k} \cdot mMov \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} mov_{i,j,k} = Amov_{j,k} \quad \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \geq op_{j,k} \quad \forall j \in A, \forall k \in P$$

Assigning Airports to Remote Tower Modules—Constraints

1. Number of airports assigned to one module $\leq mA$

2. Total number of movements within a module $\leq \max Mov$

$$\sum_{j \in A} period_{i,j,k} \leq RTM_{i,k} \cdot mA \quad \forall i \in R, \forall k \in P$$

$$\sum_{j \in A} mov_{i,j,k} \leq mMov \quad \forall i \in R, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \leq 1 \quad \forall j \in A, \forall k \in P$$

$$mov_{i,j,k} \leq period_{i,j,k} \cdot mMov \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} mov_{i,j,k} = Amov_{j,k} \quad \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \geq op_{j,k} \quad \forall j \in A, \forall k \in P$$

Assigning Airports to Remote Tower Modules—Constraints

1. Number of airports assigned to one module $\leq mA$

2. Total number of movements within a module $\leq \max Mov$

3. One airport assigned to only one module

$$\sum_{j \in A} period_{i,j,k} \leq RTM_{i,k} \cdot mA \quad \forall i \in R, \forall k \in P$$

$$\sum_{j \in A} mov_{i,j,k} \leq mMov \quad \forall i \in R, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \leq 1 \quad \forall j \in A, \forall k \in P$$

$$mov_{i,j,k} \leq period_{i,j,k} \cdot mMov \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} mov_{i,j,k} = Amov_{j,k} \quad \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \geq op_{j,k} \quad \forall j \in A, \forall k \in P$$

Assigning Airports to Remote Tower Modules—Constraints

1. Number of airports assigned to one module $\leq mA$
2. Total number of movements within a module $\leq \max Mov$
3. One airport assigned to only one module
4. All scheduled traffic from 5 airports is handled

$$\sum_{j \in A} period_{i,j,k} \leq RTM_{i,k} \cdot mA \quad \forall i \in R, \forall k \in P$$

$$\sum_{j \in A} mov_{i,j,k} \leq mMov \quad \forall i \in R, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \leq 1 \quad \forall j \in A, \forall k \in P$$

$$mov_{i,j,k} \leq period_{i,j,k} \cdot mMov \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} mov_{i,j,k} = Amov_{j,k} \quad \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \geq op_{j,k} \quad \forall j \in A, \forall k \in P$$

Assigning Airports to Remote Tower Modules—Constraints

1. Number of airports assigned to one module $\leq mA$
2. Total number of movements within a module $\leq \max Mov$
3. One airport assigned to only one module
4. All scheduled traffic from 5 airports is handled
5. All opening hours at 5 airports are covered

$$\sum_{j \in A} period_{i,j,k} \leq RTM_{i,k} \cdot mA \quad \forall i \in R, \forall k \in P$$

$$\sum_{j \in A} mov_{i,j,k} \leq mMov \quad \forall i \in R, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \leq 1 \quad \forall j \in A, \forall k \in P$$

$$mov_{i,j,k} \leq period_{i,j,k} \cdot mMov \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} mov_{i,j,k} = Amov_{j,k} \quad \forall j \in A, \forall k \in P$$

$$\sum_{i \in R} period_{i,j,k} \geq op_{j,k} \quad \forall j \in A, \forall k \in P$$

Assigning Airports to Remote Tower Modules–Objectives

$$\min \sum_{i \in R} \sum_{k \in P} RTM_{i,k}$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{l,j,k} - \sum_{j \in A} mov_{m,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{m,j,k} - \sum_{j \in A} mov_{l,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$\min \sum_{k \in P} d_{l,m,k} \quad \forall l, m \in R : l \neq m$$

$$switch_{i,j,k} \geq s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$switch_{i,j,k} \geq -s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$s_{i,j,k} = period_{i,j,k+1} - period_{i,j,k}$$

$$\min \sum_{i \in R} \sum_{j \in A} \sum_{k=1}^{p-1} switch_{i,j,k}$$

Assigning Airports to Remote Tower Modules–Objectives

1. Minimize the number of remote tower modules in use

$$\min \sum_{i \in R} \sum_{k \in P} RTM_{i,k}$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{l,j,k} - \sum_{j \in A} mov_{m,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{m,j,k} - \sum_{j \in A} mov_{l,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$\min \sum_{k \in P} d_{l,m,k} \quad \forall l, m \in R : l \neq m$$

$$switch_{i,j,k} \geq s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$switch_{i,j,k} \geq -s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$s_{i,j,k} = period_{i,j,k+1} - period_{i,j,k}$$

$$\min \sum_{i \in R} \sum_{j \in A} \sum_{k=1}^{p-1} switch_{i,j,k}$$

Assigning Airports to Remote Tower Modules–Objectives

1. Minimize the number of remote tower modules in use
2. Balance workload between the modules

$$\min \sum_{i \in R} \sum_{k \in P} RTM_{i,k}$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{l,j,k} - \sum_{j \in A} mov_{m,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{m,j,k} - \sum_{j \in A} mov_{l,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$\min \sum_{k \in P} d_{l,m,k} \quad \forall l, m \in R : l \neq m$$

$$switch_{i,j,k} \geq s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$switch_{i,j,k} \geq -s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$s_{i,j,k} = period_{i,j,k+1} - period_{i,j,k}$$

$$\min \sum_{i \in R} \sum_{j \in A} \sum_{k=1}^{p-1} switch_{i,j,k}$$

Assigning Airports to Remote Tower Modules–Objectives

1. Minimize the number of remote tower modules in use
2. Balance workload between the modules
3. Minimize assignment switches

$$\min \sum_{i \in R} \sum_{k \in P} RTM_{i,k}$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{l,j,k} - \sum_{j \in A} mov_{m,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{m,j,k} - \sum_{j \in A} mov_{l,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$\min \sum_{k \in P} d_{l,m,k} \quad \forall l, m \in R : l \neq m$$

$$switch_{i,j,k} \geq s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$switch_{i,j,k} \geq -s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$s_{i,j,k} = period_{i,j,k+1} - period_{i,j,k}$$

$$\min \sum_{i \in R} \sum_{j \in A} \sum_{k=1}^{p-1} switch_{i,j,k}$$

Assigning Airports to Remote Tower Modules–Objectives

1. Minimize the number of remote tower modules in use

2. Balance workload between the modules

As much as possible!

3. Minimize assignment switches

$$\min \sum_{i \in R} \sum_{k \in P} RTM_{i,k}$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{l,j,k} - \sum_{j \in A} mov_{m,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$d_{l,m,k} \geq \sum_{j \in A} mov_{m,j,k} - \sum_{j \in A} mov_{l,j,k} \quad \forall l, m \in R, \forall k \in P$$

$$\min \sum_{k \in P} d_{l,m,k} \quad \forall l, m \in R : l \neq m$$

$$switch_{i,j,k} \geq s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$switch_{i,j,k} \geq -s_{i,j,k} \quad \forall i \in R, \forall j \in A, \forall k \in P$$

$$s_{i,j,k} = period_{i,j,k+1} - period_{i,j,k}$$

$$\min \sum_{i \in R} \sum_{j \in A} \sum_{k=1}^{p-1} switch_{i,j,k}$$